Competing Gatekeepers

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Abstract

We extend the Baye and Morgan (2001) model to study competition between price comparison sites in the information market on the internet. We identify one symmetric sub-game perfect Nash equilibrium in which (1) price comparison sites set the same advertising fees; (2) the same proportion of consumers subscribe to each site; (3) each firm mixes between advertising on all sites and not advertising; and (4) advertised prices are dispersed. The introduction of additional price comparison sites may reduce social welfare and joint profits of price comparison sites. In the equilibrium with each consumer subscribing to one site, as the number of price comparison sites goes to infinity, the information market approaches that without price comparison sites.

KEYWORDS: information gatekeeper, internet shopping, price comparison

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1 Introduction

In today’s e-commerce, price comparison sites enhance transaction opportunities between firms and consumers by directing consumers to different firms who advertise their products on the sites.\(^1\) Usually free of charge, consumers can easily get a list of prices for a product from multiple firms while firms usually pay advertising fees to be included in online price listings.

Products on price comparison sites include books, CDs, electronics, mortgages, software, etc. Moreover, in many product categories, consumers have multiple options to choose from. For example, to find the best price for *Harry Potter and the Order of the Phoenix*, consumers may use price comparison sites such as AddAll.com, AllBookStores.com, or isbn.nu. To find the best mortgage rates, consumers can use BestRate.com, LendingTree.com, or MoneyNest.com. To find the cheapest prescription drugs, consumers can use DestinationRx.com or PharmacyChecker.com. While these price comparison sites specialize in products, other sites, such as Nextag.com, PriceGrabber.com, or priceSCAN.com, provide price information for a wide range of products.

We look at competition between price comparison sites in the information market on the internet. Baye and Morgan (2001) (BM henceforth), the basis for this research, introduce a model in which a single price comparison site, the “information gatekeeper,” interacts with consumers and firms in this information market.\(^2\) They find that only dispersed price equilibria exist in such a model. Moreover, the gatekeeper’s profits are maximized in a dispersed price equilibrium where all consumers subscribe to the gatekeeper’s services. Finally, the gatekeeper has incentives to set high advertising fees to firms while low subscription fees for consumers.

In the BM model, only one price comparison site is allowed because the focus is on the interaction between information markets and the associated product markets. However, as demonstrated by the examples above, the coexistence of multiple price comparison sites seems more common than dominance by a monopoly site.

We study competition in the information market by extending the BM model to allow multiple price comparison sites. We find equilibria in which

\(^1\) In August 2003, more than 21 million customers visited price comparison sites, according to Nielsen/Net-Ratings. The most popular sites have millions of visits. For example, Shopping.com (formerly DealTime.com) had nearly 12 million unique visits, BizRate.com had 6 million, NexTag.com had 4.6 million, and PriceGrabber.com had 3.9 million. Source: Clickz.com on Shopping Search Engines by Sherman (2003).

\(^2\) Baye and Morgan (2001) term the price comparison site an “information gatekeeper” as it charges consumers and firms fees for obtaining from it price information.
multiple sites coexist in the market and make positive profits. Social welfare may be lower with competing price comparison sites as it reduces the effectiveness of the information market.

We start out with a three-stage game model in which there are multiple price comparison sites. In stage 1, price comparison sites simultaneously set advertising fees. In stage 2, consumers and firms simultaneously make subscription and advertising decisions. Finally, in stage 3, consumers make shopping decisions. Restricting subscription fees to zero, price comparison sites compete on advertising fees to attract firms to advertise on their sites.

We discuss only symmetric equilibria, in which each firm adopts the same pricing strategy and advertises on the same number of sites when they do advertise, and each consumer subscribes to the same number of sites when they subscribe. Different types of strategy profiles can be identified as symmetric equilibria for the second stage.

We focus on the type of equilibria in which each consumer subscribes to only one site and each firm mixes between advertising on all sites and not advertising. We require that in stages 2 and 3 consumers and firms play according to this type of equilibrium as long as it exists. Given this restriction, we identify one symmetric sub-game perfect Nash Equilibria (SPNE) for the whole game. In this symmetric SPNE, multiple sites coexist in the market and make positive profits.

Compared with the model with only one price comparison site, the SPNE with each consumer subscribing to one site reduces social welfare. This is because the information market becomes less effective as firms advertise less intensively with multiple sites. The joint profits of price comparison sites are lower, but each firm earns higher expected profits with multiple sites than with only one monopoly site.

As the number of price comparison sites tends to infinity, the information market approaches the situation without any price comparison sites. Social welfare decreases to its lowest, joint profits of price comparison sites tend to zero, and each firm simply charges the monopoly price and sells to its local consumers only.

2 Literature and Stylized Facts

This paper is related to models on “temporal price dispersion.”\(^3\) That is, a firm constantly changes its posted price so consumers and rival firms are not able to predict its pricing strategies. Papers by Rothensal (1980), Varian (1980),

\(^3\)Stigler (1961) is the first on modelling price dispersion.
and Narasimhan (1988) fit into this category. Rothensal (1980) prevents firms from charging different prices to captive consumers versus non-captive consumers, leading to price dispersion. Varian (1980) has two types of consumers—informed and uninformed. Uninformed consumers purchase from a randomly selected store as long as the price charged is less than the reservation price. Informed consumers, on the other hand, purchase from the store charging the lowest price. Price dispersion arises when firms cannot price discriminate. Narasimhan (1988) separates the total mass of consumers into brand loyal consumers and brand switchers. Price dispersion arises because “firms fluctuate their prices to induce brand switchers to buy their products while at the same time minimizing the loss of profits from their loyal consumers (Narasimhan (1988), p.428).”

While these papers have examined price dispersion in conventional retail markets, others have examined price dispersion in online markets. Janssen and Moraga-González (2000), for example, adopt a search model to model price dispersion in online markets. Two consumer types differ in search costs—informed consumers have zero search costs while less-informed consumers have positive search costs. Three types of equilibria emerge with varying levels of consumers’ search intensity. Baye and Morgan (2001) model price dispersion as a single price comparison site interacting with consumers and firms in the internet information market. The main reason for a dispersed price equilibrium is that firms have to pay advertising fees to use the price comparison site’s services. By randomizing advertised prices, firms avoid marginal cost pricing and hence sustains the value of the information market.

Two main features distinguish the BM model from the other models. First, all players’ strategies are endogenous. Thus, firms decide whether to advertise on the site, consumers decide whether to subscribe to the price comparison site’s services, and the price comparison site decides on the fees it charges. Second, the model explicitly considers online market characteristics, especially the shopping tool that is becoming more and more popular among consumers—price comparison sites. By using price comparison sites, consumers obtain various prices for the same product. Thus, the search cost is basically zero for consumers.

Our paper is also related to the burgeoning literature on two-sided markets, with consumers on one side of the market, firms on the other side, and price comparison sites as platforms that court the two sides. This strand of literature mainly focus on how the platform designs price structures to resolve

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4For an introduction on two-sided markets, see Rochet and Tirole (2004). See also Rochet and Tirole (2003).
non-internalized externalities between the two sides of the market.

**Some Stylized Facts:** The results of some searches for the same product on different price comparison sites provide a picture of how firms actually price on different price comparison sites as well as a background for the model in Section 3. First, price dispersion is prevalent, as is documented by recent empirical studies. Second, some firms advertise on multiple sites at the same time. Third, when a firm advertises on multiple sites, it usually charges the same price on each site. For example, to search for the lowest price deal for Canon PowerShot S500 digital camera, we consult price listings of two sites — cnet.shopper.com and NexTag.com. The search result on October 22, 2004 is as follows. First, price dispersion is prevalent on both sites. On NexTag.com, prices range from $304.83 to $409.00, while on cnet.shopper.com, prices range from $299.95 to $355.99. Second, some firms advertise on both sites. Among those 61 firms who advertise on cnet.shopper.com, and those 64 who advertise on NexTag.com, there are 15 firms who advertise on both sites. Moreover, a more careful examination shows that when a firm advertises on both sites, it advertises the same price. There is no exception among those 15 firms in this example, and no exception among our casual searches. This paper does not intend to explain why a firm charges the same price on different price comparison sites. However, based on the observations, later in our model we will make the assumption that when a firm advertises on multiple sites, it advertises the same price.

The rest of the paper is organized as follows. Section 3 describes the model. Section 4 considers Nash equilibrium when each consumer subscribes to only one site. Section 5 discusses price comparison sites’ fee-setting decisions.

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5Empirical evidence suggests price dispersion exists in online markets. For example, Bailey (1998) finds that price dispersion is higher online than in traditional outlets, while Brynjolfsson and Smith (2000a) find mixed results for the level of price dispersion in online markets versus conventional markets for books and CDs, depending on whether prices are weighted by market share. Clay et al. (2001) and Clemons et al. (2002) find substantial price dispersion for online bookstores and airplane tickets offered by online travel agents respectively. Scholten and Smith (2002) find that price dispersion persists over 24 years periods for both retail and e-tail markets. Recently, a series of papers by Baye et al. (2004a, 2004b, 2006) also provide evidence of persistent price dispersion on the internet.

6Since shipping costs are sometimes not shown, we refer to the base price here.

7This may be justified by the following reasons. First, by advertising the same price on two different sites, firms establish goodwill based on their pricing strategies. Second, charging the same price prevents the possibility of arbitrage by consumers if they see different prices posted on different sites.

8Recently, Lin and Scholten (2005) examine pricing behaviors of firms using data collected from Cnet.shopper.com and Nextag.com. They find that after adjusting prices for rebates, more than 90% of firms who advertise on both sites charge the same price.
restricting consumers and firms to play according to the equilibrium as in Section 4. Section 6 compares social welfare of multiple price comparison sites with that of a monopoly site. Section 7 concludes.

3 The Model

Consider an information market served by \( k \) \((k \geq 2)\) price comparison sites. These \( k \) sites interact with other players – a unit mass of consumers and \( n \) firms in the information market, where \( n \geq k \). Price comparison sites charge firms “advertising fees” for posting their prices, but do not charge consumers for browsing price information. Let \( \phi_s, s = 1, 2, ..., k \), be the advertising fees charged by each site. The cost of setting up one price comparison site is \( K \).

A unit mass of consumers are evenly distributed across \( n \) geographic locations. Each location is assumed to be far from the other locations. The demand function \( q(p) \) characterizes consumer preferences for the good. It is assumed to be continuous and non-increasing in the price level, and is identical for each consumer. Consumer surplus at price \( p \) is defined to be \( S(p) \equiv \int_{p}^{\infty} q(t)dt \).

Each location is served by a single firm \( i, i = 1, 2, ..., n \). Without the internet, consumers will not travel to other locations to buy the good, as the cost of travelling is assumed prohibitive. Thus, each firm enjoys monopoly power over its local market without price comparison sites. All firms produce the good with the same technology with constant marginal cost \( c (c \geq 0) \) and no fixed costs. Define a firm’s expected profits when it sells to the whole market as \( \pi(p) = (p - c)q(p) \). Assume that there is a unique profit-maximizing price \( r \in (c, \infty) \), and that \( \pi \) is strictly increasing up to \( r \). The cost for a consumer to physically visit her local store is \( \varepsilon \), where \( \varepsilon \) is sufficiently small such that \( S(r) > \varepsilon \); i.e., it is worthwhile to physically visit the local store even if a consumer is sure that she will be charged the monopoly price.

Since consumers cannot travel to other locations, if no price comparison sites are present, each firm simply charges the monopoly price, sells to its local customers, and earns profits of \( (r - c)q(r)/n \), or \( \pi(r)/n \). On the other hand,

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9See Caillaud and Jullien (2003) for an alternative model on intermediary competition.
10This assumption – subscription is free for consumers – simplifies the analysis. This is a reasonable approximation as consumers usually pay nothing when browsing a price comparison site. Allowing for non-zero subscription fees will substantially complicate the analysis, and the discussions are beyond the scope of this paper. The type of equilibria we focus on in Section 4 and Section 5 remain valid even if we allow price comparison sites to charge non-zero subscription fees.
11Alternatively, we can interpret consumers residing in each location as consumers who are loyal to a particular firm.
with price comparison sites, firms can make their prices known to non-local consumers, and consumers can access price information of non-local firms and purchase from one of the firms who list their prices on the site(s). In fact, BM show that with one information gatekeeper, the economy can coordinate on an equilibrium in which firms do advertise with the information gatekeeper, and consumers do subscribe to its services.\footnote{As pointed out in BM, there is also an equilibrium in which firms and consumers simply ignore the information gatekeeper.}

Firms may advertise with one or more than one site, or not advertise at all. We make the following assumption about a firm’s advertising strategies:

**Assumption 1.** *Firms cannot price discriminate among different consumers.*

In other words, no matter whether a consumer reaches a firm through the site(s) that she subscribes to, or a local visit, the firm must charge the consumer the same price.\footnote{Nahm (2003) examines a monopoly site’s optimal fee structure when firms can price discriminate between consumers who consult the price comparison site and those who do not. He shows that when firms can price discriminate, the monopoly site’s optimal advertising fee is zero and firms’ advertised prices are not dispersed. Similar results may arise if we allow firms to price discriminate among different consumers when there are more than two sites. See also Baye and Morgan (2002) for discussions on the effects of price discrimination in the BM model. For empirical evidence documenting price discrimination or non-discrimination in online versus offline markets, see Baye et al. (2004a) and Ellison and Ellison (2004).}

We discuss only symmetric equilibria, in which each firm advertises on the same number of site(s) when they advertise and each adopts the same pricing strategy.\footnote{BM anticipated one asymmetric equilibrium by considering a situation in which there is an incumbent gatekeeper and a potential entrant in the information market. Equilibria with only one monopoly site hold if one of the two sites is completely ignored by consumers (or firms), because in that instance firms (or consumers) will have no incentive to switch to the other site. Using the same argument as that of BM, equilibria in which any subset \( x \) of the \( k \) sites, \( 1 \leq x < k \), are completely ignored by firms and consumers can be supported as an equilibrium. Other types of asymmetric equilibria may be obtained, for example, by allowing heterogeneity among distribution of prices when firms advertise on price comparison sites.} Denote \( l \) as the number of site(s) a firm advertises on when it advertises. Throughout this paper, we will adopt the following notation: Firm \( i \)'s \((i = 1, 2, \ldots, n)\) expected profits when it advertises a price \( p \) on at least one site and when it does not advertise are denoted as \( E\pi_i(p, l \text{ site(s)}) \) and \( E\pi_i(p, N) \), respectively. Consumers can choose to subscribe to one site, more than one site, or none. We consider symmetric equilibria in which each consumer subscribes to the same number of sites when they subscribe. Also, we consider only pure consumer strategies.

The timing of the game is summarized as follows.
Stage 1: Price comparison sites announce advertising fees $\phi_i$, $i=1,2,...,k$, simultaneously.

Stage 2: Learning these fees, consumers decide whether or not to subscribe, and if they do, which site(s) to subscribe to; simultaneously, firms make pricing decisions and decide whether or not to advertise their prices on the sites, and if they do, which site(s) to advertise on.

Stage 3: Consumers shop. Consumers decide where to purchase the good and how much to purchase. Also, once a subscribing consumer sees prices advertised on a price comparison site, she can always go back to the site to make a purchase even if she visits her local firm.

Solving backwards, we first characterize consumers’ shopping behavior. Then we find Nash equilibria for firms and consumers given advertising fees set by price comparison sites. Finally, we solve for price comparison sites’ optimal advertising fees.

Before characterizing the equilibria, notice that for any advertising fees, there always exists an equilibrium in which firms do not advertise and consumers do not subscribe (inactive market). Also, we ignore cases in which at least one site sets zero advertising fees. If we allowed an entry stage at the beginning of the game, then a rational price comparison site would not have an incentive to enter the market since its anticipated revenue is zero.

In the following section, however, we focus on Nash equilibria when each consumer subscribes to one site. Restricting consumers and firms to play according to this type of equilibria as long as they exist, we identify a symmetric SPNE in section 5. In section 6, we compare social welfare of this SPNE with that when there is one monopoly site and show that social welfare is lower with multiple price comparison sites.

4 Nash Equilibria When Each Consumer Subscribes to One Site

We consider the situation when each consumer subscribes to only one site. Recall that $k$ is the total number of price comparison sites. We divide the total mass of consumers into $k$ fractions, with each fraction representing a positive mass of consumers subscribing to each site. More formally, let $\mu_s$, $s = 1,2,...,k$, be the fraction of consumers subscribing to site $s$. For each site $s$, $\mu_s$ is positive, and $\sum_{s=1}^{k} \mu_s = 1$. Throughout Section 4, we shall assume the following:
Assumption 2. A consumer’s subscription decision is independent of her location.

Without assumption 2, different firms may have different incentives when choosing their advertising strategies. For example, all else equal, a firm with more of its local consumers subscribing to a particular site $s$ tends to have less incentives to advertise on $s$ than a firm with less of its local consumers subscribing to site $s$. This is because the main purpose of advertising is to attract consumers from other locations. Specifically, we assume that for each firm $i$, $i = 1, 2, ..., n$, a fraction $\mu_s$ of its local consumers subscribe to site $s$, $s = 1, 2, ..., k$.

We first establish the following Lemma.

Lemma 1. When each consumer subscribes to only one site, and each site has a positive mass of consumers, firms must advertise on all sites if they advertise.

The proof for Lemma 1 is omitted. An intuitive argument is provided as follows. Since subscription is free, consumers are always at least as better off subscribing than not subscribing. Thus, a consumer subscribes to a price comparison site as long as she gets information from it. Now, consider a consumer who subscribes to only one site, for example, site 1. Suppose now the probability with which a firm advertises on site 2 only is greater than zero. This implies that the consumer has a positive probability of getting different price information if she is to subscribe to site 2. She thus strictly prefers subscribing to both site 1 and site 2 to subscribing to site 1 only. The same argument applies when the probability a firm advertises on any subset of the rest $k - 1$ sites is greater than zero. Thus, when each consumer subscribes to only one site, and each site has a positive mass of consumers, firms must advertise on all sites if they advertise.

Using Lemma 1, we rule out all but two types of firms’ strategies: each firm advertises on all sites with probability one, and each firm mixes between advertising on all sites and not advertising. Before discussing these two types of firm strategies, we first describe the optimal shopping decisions by consumers.

Lemma 2. A consumer who subscribes to one site (a) first visits the site that she subscribes to and (b) purchases at the lowest price on the site. (c) If there are no price listings on the site, she visits and purchases from her local firm.

Proof. Part (a) and part (c) are obtained using the same reasoning as those in Proposition 1 of BM. Now we describe the proof for part (b). Consider a consumer who subscribes to site $s$ only. Clearly, if she observes her local firm’s
price on site \( s \), she purchases at the lowest price available on the site. If she does not observe her local firm’s price, she knows that her local firm does not advertise (by Lemma 1), and she must decide whether to incur \( \varepsilon \) and visit her local firm. The rest of the arguments follow Proposition 1 of BM.

Now taking advertising fees set by price comparison sites in the first stage as given, we would like to find Nash equilibria for the second stage simultaneous move game between firms and consumers. We first fix consumers’ subscription decisions and check for the optimality of firms’ advertising strategies. Then, given firms’ advertising strategies, we check for the optimality of consumers’ subscription decisions. First, we describe firms’ pricing decisions when they do not advertise.

**Lemma 3.** Given that each consumer subscribes to only one site, and each site has a positive mass of consumers, a firm charges the monopoly price when it does not advertise.

**Proof.** Lemma 3 is obtained using the same reasoning as those in Proposition 2 of BM.

We consider the following two cases and establish a class of equilibrium under Case 2.

**CASE 1:** Suppose that each firm advertises on all sites with probability one. This cannot be an equilibrium because Bertrand competition arises and profits (gross of advertising fees) will be driven to zero. Since the advertising fees (\( \phi_s, s = 1, 2, ..., k \)) are positive, a firm would rather not advertise because if it does, it earns negative net profits, while if it does not, its profits are at least zero.

**CASE 2:** Suppose that each firm mixes between advertising on all sites and not advertising. We first establish the following Lemma.

**Lemma 4.** When each consumer subscribes to one site, and each site has a positive mass of consumers, in an equilibrium with each firm mixing between advertising on all sites and not advertising, the ratio between a site’s advertising fee and the proportion of consumers who subscribe to it has to be equal across all sites. That is,

\[
\frac{\phi_s}{\mu_s} = \frac{\phi_t}{\mu_t}, \forall s, t \in \{1, 2, ..., k\}.
\]

**Proof.** See the appendix for the proof of Lemma 4.
Lemma 4 ensures that, in equilibrium, a firm that advertises on all sites will not have incentives to deviate to advertising on less than all sites. For example, suppose the ratio between site $s$’s advertising fee and the proportion of consumers who subscribe to it is larger than those for the rest of the price comparison sites. This means that site $s$ charges an advertising fee that is too high compared to the fraction of subscribing consumers. Then a firm would rather not advertise on site $s$ because it is not profitable to spend an extra advertising fee ($\phi_s$) and advertise on the site.

Let $\beta$ be the probability a firm advertises on all sites, $1 - \beta$ be the probability a firm does not advertise, and let $H$ be the distribution function of a firm when it advertises on all sites simultaneously. We have the following proposition.

Proposition 1. Given advertising fees ($\phi_s$, $s = 1, 2, \ldots, k$) set by price comparison sites, a symmetric Nash Equilibrium with each consumer subscribing to one site exists if and only if $0 < \sum_{s=1}^{k} \phi_s < \frac{n-1}{n} \pi(r)$ and can be characterized as follows:

1. A fraction $\mu_s$, $s = 1, 2, \ldots, k$, of consumers subscribe to site $s$, $\sum_{s=1}^{k} \mu_s = 1$, $\mu_s \neq 0$, and $\frac{\phi_s}{\mu_s} = \frac{\phi_t}{\mu_t}, \forall s, t \in \{1, 2, \ldots, k\}$;

2. Each firm advertises on all sites with probability

$$\beta = 1 - \left[\frac{n \sum_{s=1}^{k} \phi_s}{(n - 1) \pi(r)}\right] \frac{1}{n-1}, \beta \in (0, 1);$$

3. The distribution function of a firm’s advertised price is given by the c.d.f.

$$H(p) = \frac{1}{\beta} \left[1 - \left(\frac{1 - \beta}{n-1}\right) \pi(r) + \frac{n \sum_{s=1}^{k} \phi_s}{n \pi(p)} \right] \frac{1}{n-1}$$

with support $[p_k, r]$, where

$$p_k = \pi^{-1}\left[\frac{n}{n-1} \sum_{s=1}^{k} \phi_s\right];$$

and

4. Each firm earns expected profits of

$$E\pi_i = \frac{\sum_{s=1}^{k} \phi_s}{n-1}.$$
The proof of Proposition 1 is relegated in the appendix. Notice that consumers’ strategies are optimal. Since subscription is free for consumers, subscribing always weakly dominates not subscribing. Moreover, the expected surplus for a consumer from subscribing to one site is the same as that from subscribing to more than one site. This is because once firms advertise, they advertise on all sites. By subscribing to one site only, consumers get the same information as they would if they subscribed to more than one site.

5 Fee Setting Decisions by Price Comparison Sites

In the first stage, price comparison sites set advertising fees simultaneously to maximize expected profits. Now, given any set of advertising fees set by price comparison sites in the first stage, in the second stage we require consumers and firms to play according to the equilibrium as in Proposition 1 as long as this type of equilibrium exists. Given this restriction, there is only one symmetric SPNE. This SPNE could be obtained by solving the price comparison sites’ maximization problems simultaneously.

Notice that price comparison sites derive revenues entirely from advertising fees. Thus, site s’s problem is to set $\phi_s$ to maximize its profits $\Pi_s = n\beta\phi_s$, where $s = 1, 2, ..., k$.\(^{15}\) Substituting $\beta = 1 - \frac{\sum_{s=1}^{k} \phi_s}{(n-1)\pi(r)}\frac{1}{n-1}$ into price comparison sites’ profit functions and solving their maximization problems simultaneously gives the equilibrium advertising fees

$$\phi_1^* = \phi_2^* = ... = \phi_k^* = \frac{(n-1)\pi(r)}{nk} \frac{k(n-1)}{k(n-1)+1}^{n-1}.$$

Notice that we have not imposed symmetry in obtaining the equalized equilibrium advertising fees. By Lemma 4 and the assumption that $\sum_{s=1}^{k} \mu_s = 1$, we get the equilibrium fractions of consumers who subscribe to each site to be

$$\mu_1^* = \mu_2^* = ... = \mu_k^* = \frac{1}{k}.$$

We have the following Proposition.

**Proposition 2.** Suppose in the second stage we require firms and consumers to play according to the equilibrium in Proposition 1 as long as this type of equilibrium exists. Then there is only one symmetric SPNE for the whole game. This SPNE could be characterized as follows.

\(^{15}\)The setup cost $K$ is not included in site s’s profit function since it is a sunk cost at the post-entry stage.
(1) In stage 1, price comparison sites set advertising fees $\phi_1^* = \phi_2^* = \ldots = \phi_k^* = \frac{(n-1)\pi(r)}{nk} \cdot \frac{k(n-1)}{(k(n-1)+1)}n-1$ simultaneously.

(2) In stage 2, consumers and firms play according to the Nash equilibrium in Proposition 1 after they learn advertising fees set in stage 1; The proportions of consumers subscribing to each site satisfy $\mu_1^* = \mu_2^* = \ldots = \mu_k^* = \frac{1}{k}$.

(3) In stage 3, consumers shop in accordance with Lemma 2.

6 Welfare Analysis

Before making the welfare comparison, we first show the following lemma.

Lemma 5. The joint profits of price comparison sites are lower than the profits of one monopoly site.

Proof. See the appendix for the proof of Lemma 5.

Lemma 5 is driven by the fact that the joint advertising fees charged by the $k$ sites, i.e., $\sum_{s=1}^{k} \phi_s^*$, are larger than the advertising fee charged by a monopoly site, i.e., $\phi^*$ in the BM model. The amount of joint advertising fees, $\sum_{s=1}^{k} \phi_s^*$, is equal to $k\phi_1^*$ since each site charges the same advertising fee in equilibrium. The advertising fee charged by a monopoly site can be obtained by setting the subscription fee to be zero in the BM model and solving for the monopoly site’s maximization problem to get

$$\phi^* = \left(\frac{n-1}{n}\right)^n \pi(r)$$

Comparing $k\phi_1^*$ with $\phi^*$, we see that $k\phi_1^* > \phi^*$ since $\frac{n-1}{n} < \frac{k(n-1)}{k(n-1)+1}$.

Now we explain Lemma 5. Notice that joint expected profits of price comparison sites are maximized when each site sets its advertising fee to be $\phi^*/k$. However, given that other sites all set their advertising fees to be $\phi^*/k$, it is not optimal for one site to also set this advertising fee. Instead, it has an incentive to set an advertising fee that is higher than the rest of the firms’.

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16 This can be seen by looking at the following two equations. Differentiating the monopoly site’s expected profits with respect to $\phi$ gives

$$n[1 - \left(\frac{n\phi^*}{(n-1)\pi(r)}\right)^{\frac{1}{n-1}}] + n\phi^*(\frac{1}{n-1})\left[\frac{n\phi^*}{(n-1)\pi(r)}\right]^{\frac{1}{n-1}-1} \cdot \frac{n}{(n-1)\pi(r)} = 0.$$  

The first term is the monopoly site’s increased profits because of an increase in the ad-
This is because when site $s$ maximizes its expected profits, it takes into account only the changes in revenue from firms who advertise on its site rather than on all sites. Thus, the joint profits of price comparison sites are lower than the profits of one monopoly site.

Next we compare social welfare of the SPNE as in Proposition 2 with that when there is only one monopoly site, i.e., the BM equilibrium. Setting $\mu$, the fraction of consumers subscribing to the gatekeeper’s services, equal to one in Proposition 3 of BM and comparing the BM equilibrium with that of Proposition 1 in this paper, we see that, when $\phi$, the advertising fee in the BM model, is equal to $k\phi_1$, the respective probability of advertising and the distribution functions are the same. Thus, by comparing the equilibrium advertising fee, $\phi^*$, in the BM model with $k\phi^*_1$ in Proposition 1, we are able to know the changes in social welfare since social welfare depends on the probability of advertising and the associated distribution function, which in turn depends on the advertising fee. We have the following Proposition.

**Proposition 3.** Social welfare and consumer surplus are both lower, while each firm’s expected profits are higher when there are multiple price comparison sites with each consumer subscribing to one site than when there is only one monopoly site.

**Proof.** First, we show that the value of social welfare is lower with multiple price comparison sites than with one monopoly site. Since $k\phi^*_1 > \phi^*$, we know the lower bound of $H$ is higher than the lower bound of $F$ for the BM model. Moreover, $k\phi^*_1 > \phi^*$ implies that $\beta < \alpha$, where $\alpha$ is the probability of advertising in the BM model. That is, in the equilibrium as described in Proposition 1, firms advertise less intensively than in the equilibrium when there is only one price comparison site. Thus, they are more likely to advertise a price that is higher than in the monopoly-site case.

The value of social welfare when there are $k$ price comparison sites could

$$n[1 - (\frac{n(\sum_{s=1}^k \phi_s)}{(n-1)\pi(r)})^{\frac{1}{n-1}}] + n\phi_s(-\frac{1}{n-1})[n(\sum_{s=1}^k \phi_s)]^{\frac{1}{n-1}} - \frac{n}{(n-1)\pi(r)} = 0.$$

Substituting $\phi_1 = \phi_2 = ... = \phi_k = \phi^*/k$ into the first order condition of site $s$, and comparing it with the first order condition of the monopoly site, we see that it is not optimal for site $s$ to set its advertising fee to be $\phi^*/k$. Instead, it has incentives to set an advertising fee that is larger than $\phi^*/k$. 

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be written as follows:

$$SW^k = \int_{p_k}^r [S(p) + \pi(p)]d[1 - (1 - \beta H(p))^n] + (1 - \beta)^n[S(r) - \varepsilon + \pi(r)] - kK,$$

where $p_k$ is the lower bound of $H$ as defined previously.

On the other hand, the value of social welfare with one monopoly site is:

$$SW^M = \int_{p_1}^r [S(p) + \pi(p)]d[1 - (1 - \alpha F(p))^n] + (1 - \alpha)^n[S(r) - \varepsilon + \pi(r)] - K,$$

where $p_1$ is the lower-bound of $F$ in the BM model.

Let $p^M$ be the final transaction price for the consumer when there is only one price comparison site, and $p^k$ be that when there are $k$ price comparison sites. Then the c.d.f. of $p^M$ has mass $(1 - \alpha)^n$ at the monopoly price $r$, and similarly the c.d.f. of $p^k$ has mass $(1 - \beta)^n$ at the monopoly price $r$. Since we know that the lower-bound of $H$ is higher than the lower-bound of $F$, that $1 - (1 - \alpha F(p))^n$ decreases with $\phi$ (by differentiating $1 - (1 - \alpha F(p))^n$ with respect to $\phi$), and that $k\phi^*_s > \phi^*$, we know the distribution function of $p^k$ first order stochastically dominates the distribution function of $p^M$. Since $S(p) + \pi(p)$ is decreasing in $p$, we know $SW^k < SW^M$.

Second, consumer surplus is higher with multiple sites than with one monopoly site. This can be explained by the first order statistical dominance of $p^k$ over $p^M$.

Finally, each firm’s expected profits are higher with multiple sites than with one monopoly site. This is because $E\pi_i = \frac{\sum_{s=1}^k \phi^s}{n-1} \phi^*_i$, and $\sum_{s=1}^k \phi^*_s = k\phi^*_i > \phi^*$.

That social welfare is lower with multiple sites than with one monopoly site is explained by changes in the advertising fee. With multiple price comparison sites, the total amount of advertising fees firms pay is higher than that paid with one monopoly site. Higher advertising fees induce firms to advertise less intensively. As a result, consumers are less likely to find a lower price deal on the price comparison sites than with one monopoly site. Since total surplus is decreasing in the price level, social welfare is lower with $k$ sites than with one monopoly site.

We next show the following Proposition.

**Proposition 4.** When there are multiple price comparison sites, in the symmetric SPNE with each consumer subscribing to one site, as the number of price comparison sites increases to infinity, social welfare decreases to its lowest, joint profits of price comparison sites decrease to zero, and each firm’s expected profits increase to the profit level without the information market.

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Proof. We first show that social welfare decreases with the number of sites. For this purpose, we need to know how \( k\phi^*_1 \) changes with respect to \( k \) as this will allow us to know how the lower bound of \( H \) and the probability of advertising change when the number of price comparison sites increases.

Since \( k\phi^*_1 = \left( \frac{n-1}{n} \right) \pi(r) \left( \frac{k(n-1)}{k(n-1)+1} \right)^{n-1} \), and \( \frac{k(n-1)}{k(n-1)+1} \) increases with respect to \( k \), we know \( k\phi^*_1 \) increases with respect to \( k \). This implies that \( p_k \) increases while the probability of advertising, \( \beta \), decreases as \( k \) increases. Moreover, we know that \( p_k \) tends to the monopoly price \( r \) as \( k \) tends to infinity. This is because \( p_k = \pi^{-1}(\frac{n}{n-1}k\phi_1) \) and \( \lim_{k \to \infty} \{k\phi_1\} = \left( \frac{n-1}{n} \right) \pi(r) \). Since the function \( 1 - (1 - \beta H(p))^n \) decreases with \( \phi \), and we know \( p_k \) is close to \( r \) as \( k \) goes to \( \infty \), social welfare with \( k \) price comparison sites, \( SW_k \), decreases with \( k \).

As \( k \to \infty \), the joint profits of price comparison sites tend to zero. This can be easily verified because price comparison sites’ joint profits are \( n\beta k\phi^*_1 \), which goes to zero as \( k \) goes to infinity because \( \beta \) goes to zero and \( k\phi^*_1 \) goes to \( \frac{n-1}{n} \pi(r) \) as \( k \) goes to infinity.

Finally, as \( k \) goes to infinity, \( \lim_{k \to \infty} E\pi_i = \lim_{k \to \infty} \{\frac{k\phi^*_i}{n-1}\} = \frac{\pi(r)}{n} \), which is the profit level without price comparison sites.

Lizzeri (1999) studies a model in which a certification intermediary can charge the seller a fee for testing its product and then decide on how much information to reveal to uninformed buyers. The role of the intermediary in Lizzeri’s model is somewhat similar to the price comparison site (or information gatekeeper) in our model. Our finding, that when the number of price comparison sites gets large, the joint profits of these sites tend to zero echoes with the result of Lizzeri (1999).\(^{17}\)

That each firm’s expected profits increase with the number of price comparison sites is consistent with the welfare analysis above. As the number of price comparison sites increases, each firm’s probability of advertising becomes lower as a result of increased advertising fees. As \( k \) goes to infinity, the information market approaches the situation without price comparison services and each firm simply charges the monopoly price and sells to its local consumers only.

\(^{17}\)Lizzeri (1999) shows that when there are multiple intermediaries, there is always an equilibrium in which information is fully revealed and intermediaries make zero profits. As the number of intermediaries goes to infinity, this is the only equilibrium.
7 Conclusions and Discussions

We consider a model in which multiple sites offer consumers free access to their price comparison services, and charge firms fees for advertising. We focus on symmetric equilibria and identify one SPNE by putting restrictions on the second stage Nash equilibrium played by firms and consumers. We show that in this symmetric SPNE, (1) price comparison sites set the same advertising fees; (2) the same proportion of consumers subscribe to each site; (3) each firm mixes between advertising on all sites and not advertising; (4) advertised prices are dispersed. In our model, multiple sites coexist in the market and make positive profits.

Compared with the model with one monopoly site, the equilibrium with each consumer subscribing to one site reduces social welfare. This is because the information market functions less effectively as firms advertise less intensively with multiple price comparison sites. As a result, joint profits of price comparison sites are lower while the expected profits of each firm are higher with multiple sites than with one monopoly site. As the number of price comparison sites goes to infinity, the information market approaches that without price comparison sites – social welfare decreases to its the smallest, joint profits of price comparison sites tend to zero, and each firm simply charges the monopoly price and sells to its local consumers only.

In this paper, we have focused on symmetric equilibria with each consumer subscribing to one site and showed that social welfare decreases with the number of sites. However, there may exist other types of symmetric equilibria that are also interesting. For example, it can be shown that there exist equilibria in which each consumer subscribes to all sites.\textsuperscript{18} Moreover, there may exist equilibria in which each consumer subscribes to \( m \) sites, where \( 1 < m < k \). Future research may explore different types of symmetric or asymmetric equilibria and their competition and welfare effects. Moreover, given the possibility of the richness of multiple equilibria, future research may explore criteria for the selection of the most “reasonable” equilibria.

Appendix

\textit{Proof of Proposition 1:} Part (1) of Proposition 1 follows from Lemma 4. Next, we show parts (2) and (3). If a firm advertises price \( p \) on all sites, its expected

\textsuperscript{18}In this type of equilibria, each consumer subscribes to all sites, and each firm mixes between advertising on one of the \( k \) sites and not advertising. A detailed proof is available upon request.
profits will be

\[ E\pi_i(p, k \text{ sites}) = \sum_{j=0}^{n-1} \left( \frac{n-1}{j} \right) \beta^j (1 - \beta)^{n-1-j} (1 - H(p))^j \pi(p) - \sum_{s=1}^{k} \phi_s. \] (1)

That is, firm \( i \)'s expected profits if it advertises a price \( p \) on all sites is a weighted average of its profits when \( j \) other firms also advertise on all sites. In the event exactly \( j \) out of the remaining \( n-1 \) firms advertise, firm \( i \) gets positive sales and earns profits if and only if their advertised prices are all higher than firm \( i \)'s advertised price, \( p \). Firm \( i \) pays advertising fees \( \sum_{s=1}^{k} \phi_s \) by advertising on all sites. Using the Binomial Theorem, the above expression could be written as

\[ E\pi_i(p, k \text{ sites}) = \pi(p)(1 - \beta H(p))^{n-1} - \sum_{s=1}^{k} \phi_s. \] (2)

If firm \( i \) does not advertise, it charges the monopoly price \( r \) and sells to its local consumers only when none of the remaining \( n-1 \) firms advertises. Hence, firm \( i \)'s expected profits if it does not advertise are

\[ E\pi_i(r, N) = (1 - \beta)^{n-1} \frac{\pi(r)}{n}. \] (3)

Setting (2) to be equal to (3) at \( p = r \) and imposing the condition that \( H(r) = 1 \), we get a firm’s probability of advertising:

\[ \beta = 1 - \frac{\left[ \frac{n}{(n - 1)\pi(r)} \right]^{\frac{1}{n-1}}}{\phi_s}. \] (4)

Notice that \( \beta \in (0, 1) \) whenever \( \sum_{s=1}^{k} \phi_s \in (0, \frac{n-1}{n} \pi(r)) \), which is the if and only if condition on the sum of advertising costs.

Equating (2) and (3) and solving for \( H \) gives

\[ H(p) = \frac{1}{\beta} \left\{ 1 - \left[ \frac{(1 - \beta)^{n-1} \pi(r) + n(\sum_{s=1}^{k} \phi_s)}{n\pi(p)} \right]^{\frac{1}{n-1}} \right\}. \] (5)

To show that \( H \) is part of an equilibrium, we need to show the following. First, \( H \) is an atomless distribution with support \([p_k, r]\). The lower support of \( H \), \( p_k \), could be obtained by setting \( H(p_k) = 0 \) and solving to get

\[ p_k = \pi^{-1}\left[ \frac{n}{n - 1}(\sum_{s=1}^{k} \phi_s) \right]. \]
Since $\frac{n}{n-1}\left(\sum_{s=1}^{k} \phi_s\right) > 0$, $\pi(p_k) = \frac{n}{n-1}\left(\sum_{s=1}^{k} \phi_s\right) > 0$ and hence $p_k > c$. Also, since $\sum_{s=1}^{k} \phi_s < \frac{n}{n-1}\pi(r)$, we know $p_k = \pi^{-1}\left[\frac{n}{n-1}\left(\sum_{s=1}^{k} \phi_s\right)\right] < r$. Second, $H$ is increasing. This is because $\pi(\cdot)$ is continuous and increasing up to $r$. Finally, it can be shown that a firm earns strictly lower expected profits if it deviates to advertising a price outside the support of $H$ when the rest $n-1$ firms all advertise according to $H$.

Part (4) of Proposition 1 can be obtained by substituting (4) into (3).

Proof of Lemma 4: Lemma 4 ensures that in equilibrium, a firm who advertises on $k$ sites will not deviate to advertising on less than $k$ sites. In the following discussion, we will derive the condition which ensures that a firm will not deviate to advertising on $k-1$ sites. The same condition that prevents a firm from deviating to advertising on less than $k-1$ sites can be derived by using the same procedure.

Given the remaining $n-1$ firms all mix between not advertising and advertising on $k$ sites, if firm $i$ deviates to advertising a price $p$ on sites 1, 2, ..., $k-1$, its expected profits will be

$$E\pi_i(p, \text{ sites } 1, 2, ..., k-1) = \sum_{j=1}^{n-1} \binom{n-1}{j} \beta^j (1-\beta)^{n-1-j} (1-H(p))^j \sum_{s=1}^{k-1} \mu_s \pi(p)$$

$$+(1-\beta)^{n-1} \left[\sum_{s=1}^{k-1} \mu_s \pi(p) + \frac{\mu_k}{n} \pi(p)\right] - \sum_{s=1}^{k-1} \phi_s.$$

In the above equation, the first term is the expected profits for firm $i$ when its advertised price is lower than those advertised by all of those $j$ firms ($j \geq 1$) who advertise. The second term is the expected profits for firm $i$ when none of the remaining $n-1$ firms advertises. Firm $i$ pays advertising fee $\sum_{s=1}^{k-1} \phi_s$ by advertising on sites 1, 2, ..., $k-1$ only. Using the Binomial Theorem, we can rewrite the previous equation as

$$E\pi_i(p, \text{ sites } 1, 2, ..., k-1) = \sum_{s=1}^{k-1} \mu_s \pi(p)(1-\beta H(p))^{n-1}$$

$$+(1-\beta)^{n-1} \frac{\mu_k}{n} \pi(p) - \sum_{s=1}^{k-1} \phi_s.$$

In order for a firm not to deviate and advertise on sites 1, 2, ..., $k-1$ only, the payoff for the firm from advertising on all sites has to be at least as
much as that from advertising on sites 1, 2, ..., k – 1 only. Hence, we need $E\pi_i(p, \text{ all sites}) \geq E\pi_i(p, \text{ sites 1, 2, ..., k – 1}).$

However, notice that if firm $i$ is to deviate to advertising on sites 1, 2, ..., k – 1 only, it will advertise the monopoly price $r$. The reason is as follows. If firm $i$ deviates to advertise on sites 1, 2, ..., k – 1 only, its problem is to maximize (6), i.e., the deviation profits. Equation (6) can be written as

$$E\pi_i(p, \text{ sites 1, 2, ..., k – 1}) = \sum_{s=1}^{k-1} \mu_s \cdot [E\pi_i(p, \text{ all sites}) + \sum_{s=1}^{k} \phi_s] + (1 - \beta)^{n-1} \frac{\mu_k}{n} \pi(p) - \sum_{s=1}^{k-1} \phi_s. \quad (7)$$

Since $E\pi_i(p, \text{ all sites})$ is simply the equilibrium profits and $\pi(p)$ is maximized at $r$, $E\pi_i(p, \text{ sites 1, 2, ..., k – 1})$ is maximized at $r$. The intuition is as follows. Compared with advertising on all sites, the main benefit to firm $i$ from advertising on sites 1, 2, ..., k – 1 is that when none of the remaining $n – 1$ firms advertise, firm $i$ can get profits from its local consumers who subscribe to site $k$ even though it does not advertise on site $k$. This benefit increases with the price level. Hence, if firm $i$ deviates to advertising on sites 1, 2, ..., k – 1 only, it will advertise the monopoly price $r$.

Using (4), (5), and imposing the condition that (6) is maximized at the monopoly price $r$, we get

$$\mu_k \sum_{s=1}^{k} \phi_s \geq \phi_k. \quad (8)$$

Similarly, for firm $i$ not to deviate to advertise on sites 1, 2, ..., k – 2, k and on sites 1, 2, ..., k – 3, k – 1, k, etc., we obtain the following conditions.

$$\mu_{k-1} \sum_{s=1}^{k} \phi_s \geq \phi_{k-1}, \text{ and } \mu_{k-2} \sum_{s=1}^{k} \phi_s \geq \phi_{k-2}, \text{ etc.}$$

Combining the above conditions and rearranging terms, we obtain

$$\frac{\phi_s}{\mu_s} = \frac{\phi_t}{\mu_t}, \forall s, t \in \{1, 2, ..., k\}.$$ 

**Proof of Lemma 5:** The equilibrium joint profits of price comparison sites can be written as

$$\sum_{s=1}^{k} \Pi^*_s = \pi(r) \cdot k^{n-1} \cdot \left[\frac{n-1}{k(n-1)+1}\right]^n - kK$$
while the total profits of one monopoly site can be written as

\[ \Pi^*_{M} = \left( \frac{n - 1}{n} \right)^n \pi(r) - K. \]

It can be shown that the ratio between the first term of \( \sum_{s=1}^{k} \Pi^*_s \) and the first term of \( \Pi^*_M \) is equal to \( \frac{1}{k} \cdot [1 + \frac{k-1}{k(n-1)+1}]^n \), which is less than one for all \( n \geq 2 \).

**References**


