Vertical Integration and Exclusivity in Platform and Two-Sided Markets

Robin S. Lee

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Abstract

This paper develops techniques to analyze the adoption decisions of consumers and firms for competing platform intermediaries in two-sided markets, and applies them to empirically measure the impact of vertical integration and exclusive contracting in the sixth-generation of the U.S. videogame industry (2000-2005). I first introduce a framework to structurally estimate consumer demand in platform-intermediated markets which (i) simultaneously estimates both hardware platform and software adoption decisions; (ii) accounts for dynamic issues including the durability of goods, agents' timing of purchases, and selection of heterogeneous consumers across platforms and time; and (iii) explicitly provides the marginal contribution of an individual software title to each platform's installed base of users. Demand results show that a platform provider's gains from exclusive access to certain software titles can be large, and failure to account for dynamics, consumer heterogeneity, and multiple hardware purchases significantly biases estimates. I next specify a dynamic network formation game to model the hardware adoption decisions of software providers, and use estimates to determine the new equilibrium industry structure if exclusive vertical arrangements were prohibited. Counterfactual experiments indicate that exclusivity benefited the smaller entrant platforms and not the dominant incumbent, which stands contrary to the interpretation of exclusivity as primarily a means of foreclosure and entry deterrence.

Keywords: platform competition, two-sided markets, vertical integration, exclusive contracting, dynamic demand, network formation, videogame industry

JEL Classification Numbers: C61, C63, C73, L13, L14, L42, L86
1 Introduction

In most networked industries, consumers adopt, join, or visit a platform in order to access goods or services provided by firms who are also affiliated with the same platform. Also known as platform or two-sided markets, these industries include hardware-software markets, content and media markets, retail marketplaces, payment systems, and even some forms of “buyer-seller networks.” Often certain firms and their products have significant market power over consumers, inducing consumers to adopt the platform(s) that they have joined. Via exclusive contracts or vertical integration, platforms compete fiercely with one another to get these firms exclusively “onboard” and dominate the market. This paper studies these exclusive vertical arrangements between platforms and firms and measures their impact on industry structure and competition.

Whether or not such arrangements are primarily pro- or anti-competitive or harmful to consumers is still a source of active debate and an open empirical question. On one hand, exclusive contracts raise anti-competitive issues since they may deter entry or foreclose rivals; the presence of network externalities can exacerbate these concerns. From a consumer welfare perspective, these vertical arrangements can also limit consumer choice by preventing consumers on competing platforms from accessing exclusive content, products, or services.

On the other hand, theory has argued that exclusive arrangements may also have pro-competitive benefits, such as encouraging investment and effort provision by contracting partners. In networked industries, integration by a platform provider may be effective in solving the “chicken-and-egg” coordination problem, one of the fundamental barriers to entry discussed in the two-sided market literature. Furthermore, exclusivity may be an integral tool used by entrant platforms to break into established markets: by preventing contracting partners from also supporting the incumbent, an entrant can gain a competitive advantage, spur adoption of its own platform, and thereby spark

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1See e.g., Caillaud and Jullien (2003), Evans (2003), Rochet and Tirole (2003, 2006), Armstrong (2006), and Evans, Hagiu, and Schmalensee (2006). Examples of hardware-software markets include CD and DVD players and their respective media, videogame consoles and games, and PCs and their compatible applications or peripherals (in which hardware usually refers to the CPU and operating system combination). Many standards battles also occur within these industries, in which the success of competing hardware platforms (e.g., VHS/Betamax, HD-DVD/Blu-ray) depends crucially on the provision of compatible software. Other examples of platforms include: cable/satellite television and satellite/terrestrial radio providers, internet portals and aggregators, and online music services; shopping malls, department stores and brick-and-mortar merchants who “rent” space to vendors and manufacturers; credit and debit cards (Visa, Mastercard) and electronic money systems (Paypal, FeliCa, Edy); and HMOs (which consumers join to access the services of affiliated hospitals). Online advertising exchanges (Google, Microsoft, Yahoo!) are also examples of platforms, in which the relevant parties are advertisers who join to get access to “publishers” (sites where ads are shown).


3E.g., exclusivity may induce more consumers to join a platform, which in turn encourages more firms to subsequently join, and then more consumers, and so on. Thus, an entrant platform may not be able to gain critical mass and hence be deterred from entering (Shapiro, 1999). Evans (2003) discusses other related antitrust issues brought up within two-sided markets. Accessible surveys of the network effects literature in general are Katz and Shapiro (1994), Shy (2001) and Farrell and Klemperer (2007).


5I.e., without firms, consumers will not join a platform; without consumers, firms will not join.
greater platform competition.

Given the growing prevalence of networked and platform industries, the ability to resolve this theoretical ambiguity is of central importance for policy and regulation. The competitive implications of integration and exclusive contracting were at the heart of several recent prominent antitrust cases – e.g., *U.S. v. Microsoft* [253 F.3d 34 (2001)],6 *European Union v. Microsoft* [COMP/C-3/37.792 (2004)], and *U.S. v. Visa* [344 F.3d 229 (2003)]7 – and also the central issues to consider whenever evaluating exclusive carriage deals in the media industry,8 or deciding whether or not to open up closed hardware-software systems to competitors.9

This paper has two primary goals. The first is to develop a framework for analyzing the adoption decisions of consumers and firms for competing platform intermediaries – fundamentally, an analysis of how exclusive arrangements affect the competitive structure of a networked industry requires an understanding of how parties on each side of the market choose which platform(s) to join. I develop a structural model of consumer demand which accounts for the presence of indirect network effects by explicitly specifying platform utility as a function of its affiliated products, and recover how consumer demand responds to changes in a platform’s characteristics and its contracting partners. The demand model contributes to the literature on estimating indirect network effects by allowing for consumers to have preferences over the identity and not just quantity of affiliated firms, incorporating heterogeneous consumers and their selection across platforms and time, handling dynamic concerns such as the durability of goods and agents’ timing of purchases, and permitting consumers to “multihome” and purchase multiple platforms.10 I next specify a model of industry dynamics in which firms strategically choose which platforms to join, and detail the computation of a new equilibrium when exclusive arrangements are prohibited. In this model, all firms and consumers make platform adoption decisions conditioning on both the observed and expected future state of the industry.

The second objective is to apply the framework to empirically measure the impact of exclusive vertical arrangements between platforms and firms in a canonical hardware-software market: the sixth-generation of the U.S. videogame industry (2000-2005). Comprising multiple differentiated hardware platforms each with its own distinct base of software, the videogame industry exhibits

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6See Whinston (2001) for a discussion on the Microsoft case, which involved Microsoft’s integration of Internet Explorer into Windows and the possible foreclosure of Netscape, a rival browser application provider.

7The Visa case dealt with the possible foreclosure of competing payment cards (American Express and Discover) from accessing the services of exclusive member banks; Rochet and Tirole (2002) provide an economic analysis of the payment card industry and discuss antitrust concerns.

8E.g., in 2007 Major League Baseball and satellite television operator DirecTV agreed to an exclusive deal which would deny cable television subscribers access to a package of “out-of-market” baseball games. The deal was eventually scuttled after a U.S. Senate hearing into the issue pressured MLB to renegotiate with excluded parties; MLB ultimately agreed to supply both cable and satellite TV operators (“Baseball Strikes Deal to Keep ‘Extra Innings’ Package on Cable,” Associated Press, April 4, 2007).

9E.g., France recently passed a law that allowed regulators to require Apple to open up its iTunes music service to other companies’ music players (“French iTunes Law Goes Into Effect,” Associated Press, August 8, 2006). Consumer rights organizations in other countries including Germany, Finland, and Norway as well as the European Union consumer chief have taken similar stances (“EU takes aim at Apple over iTunes,” Reuters, March 11, 2007).

10Other empirical papers on measuring the impact of indirect network effects include Saloner and Shepard (1995), Gandal, Kende, and Rob (2000), and Rysman (2004).
features easily generalizable to a variety of networked environments; indeed, two of the main hardware providers – Sony and Microsoft – are participants in several other platform markets. The videogame industry is also convenient to study since exclusive contracting and integration by platforms into software development are not intended to foreclose third-party software providers; on the contrary, these exclusive arrangements are intended to attract other software providers as much as they are intended to bring onboard additional consumers. Thus, focusing on the videogame industry can separate out potential anti-competitive effects in software development and instead focus on the possibility of foreclosure and entry-deterrence in hardware provision.

The main finding of this paper is that exclusive arrangements between hardware platforms and software publishers were pro-competitive at the platform level, and their presence benefited the smaller entrant platforms at the expense of the incumbent. Absent the ability to integrate or exclusively contract with software providers, the entrant platforms would have sold fewer hardware devices and the incumbent would have captured an even more commanding share of the market. The intuition is as follows: without exclusive arrangements, high quality software titles would have primarily developed for the incumbent due to its larger installed base, and only then (if at all) developed a version for either entrant platform; as a result, neither of the new platforms would have been able to offer consumers any significant benefit over the incumbent, and hence neither would have been able to gain substantial market share. Exclusive access to certain software titles, however, could create a competitive advantage and attract enough consumers to get a platform “off the ground,” and thus was leveraged by the entrants to gain traction in this networked industry. Indeed, the use of these exclusive arrangements may have ultimately prevented monopolization by a single hardware provider.

However, if exclusive vertical arrangements between hardware and software providers were prohibited, consumers may have benefitted from access to a greater selection of software titles onboard any given platform – absent exclusivity, counterfactual results indicate consumer welfare would have increased by approximately $7B during the five year period holding fixed the pricing and entry/exit decisions of all platform providers and software titles. This is partially a static result; whether or not consumers would have been better or worse off depends crucially on how software production would have responded to the absence of exclusive arrangements, and how the dominant platform would have reacted to its increased market power. If previously integrated “first-party” titles no longer were produced, competing platforms exited, or the incumbent raised prices, most consumer welfare gains from increased access to software would have been eliminated. This highlights the importance of accounting for dynamic effects (increased platform competition and investment) when

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11 Unlike computer applications where users typically only need one word processor, browser, or media player, videogames are not direct substitutes for one another and are instead more like “disposable” media goods which face demand for continual replacement. Thus exclusive software, by drawing onboard more consumers, increases the potential market that other software providers can access but does not necessarily crowd out or compete against these other titles. I discuss this issue further in section 3.

12 One of the “entrants” I refer to during this generation is Nintendo even though it originally entered the videogame industry in the 1980’s. This is because Nintendo and Microsoft did not introduce their sixth-generation consoles until a year after the previous generation market leader, Sony, had released its new platform and obtained a substantial lead in installed base and software titles.
addressing the potential static inefficiencies and costs (reduced access and software variety onboard each platform) of exclusivity.

When evaluating these implications in a broader context, it is important to note that “forced exclusivity” contracts – in which a software developer is not allowed to release software for a hardware platform unless it did so exclusively – are implicitly not considered when measuring the impact of exclusivity. During most of the 1980’s and 1990’s, the dominant videogame platform provider, Nintendo, forced its developers to release games exclusively using these contracts. However, following a 1992 antitrust investigation related to *Atari Games Corp v. Nintendo of America, Inc* [975 F.2d 832 (1992)], Nintendo dropped these practices. Since then, forced exclusivity contracts have not been utilized within the videogame industry, and will not be considered within the space of vertical agreements considered here.

Previous empirical work on measuring the effects of exclusive contracting and vertical integration has primarily focused on supply-side consequences and the threat of “upstream” foreclosure. In contrast, this paper focuses on “downstream” competition, and how exclusivity interacts with the networked aspect of the industry to either deter or enable platform entry. Furthermore, this paper is one of the first to explicitly account for the rematching process between contracting partners within a counterfactual regime.

### 1.1 Framework for Analysis

The paper is structured in three parts: the first two develop and apply a framework for analyzing how industry participants chose which hardware platform to purchase or develop for, and the third applies estimates obtained from the first two stages to evaluate regimes in which exclusive vertical arrangements are prohibited.

**Consumer Demand**

The first part of this paper develops and estimates a structural model of dynamic consumer demand in hardware-software markets. Although also of independent interest, the demand system is primary

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14These types of forced agreements have also been ruled to be anti-competitive in other industries: e.g., the U.S. District Court for the Southern District of New York in *U.S. v. Visa* [163 F.Sup.2d 322 (2001)] found these types of contracts to “weaken competition and harm consumers” by limiting output of rival payment card providers and foreclosing them from competing in related markets.

15The literature on vertical restraints has typically referred to an “upstream” firm as the supplier of a (possibly intermediate) good in a vertical chain, and a “downstream” firm as a firm that uses the good to produce another product, or a wholesale or retail firm that resells the good to final consumers (Tirole (1988), ch. 4). I am implicitly referencing this structure when I refer to software providers as “upstream” and hardware platforms as “downstream” entities.

16E.g., Asker (2004) and Sass (2005) analyze the relationship between manufacturers and distributors in the beer industry, and find evidence that exclusive arrangements seem to enhance efficiency, improve distributor effort, and do not foreclose smaller brewers. Chipty (2001) studies the integration between programming and distribution in the cable television industry, and finds that although certain programming may have been foreclosed from distribution by integrated cable providers, the associated efficiency gains have likely offset any social costs generated. See Lafontaine and Slade (2008) for a survey of empirical studies on vertical restraints.
used to determine how consumer demand for hardware responds to changes in software availability, and the number of copies a given title would expect to sell conditional on joining any platform. These estimates are inputs in the second part of the paper, which focuses on determining the set of platforms each software title will join.

One of the primary reasons for adopting a structural approach in demand estimation is to recover the marginal contribution of an individual software title to each platform’s installed base of users. Often consumers select which platform to purchase based on the presence of a particular “hit title” or “killer application.” In such cases when one side of the market is oligopolistic or certain firms have significant market power over consumers, analyzing the impact of exclusive arrangements requires computing how demand and firm profits are affected when specific contracting partners change.\footnote{See e.g. Lee (2006). This is similar to calculating the underlying “value” functions in multilateral contracting environments with externalities (e.g., Prat and Rustichini (2003)), partition functions in coalitional bargaining games, and graph functions in bipartite network formation games (e.g., Bloch and Jackson (2007)).} Although other papers have estimated demand systems in platform markets, most have done so by ignoring or taking a reduced form approach to one side of the market, and consequently have been unable to recover the impact of an individual title.\footnote{E.g., Nair, Chintagunta, and Dubé (2004), Clements and Ohashi (2005), Prieger and Hu (2006), Corts and Lederman (2007), and Dubé, Hitsch, and Chintagunta (2007) specify software utility on a platform as a function of only the total number of titles onboard. Notable exceptions are Town and Vistnes (2001), Capps, Dranove, and Satterthwaite (2003), and Ho (2006): these papers estimate static demand systems for managed care health organizations (such as HMOs) as a function of individual affiliated hospitals. However, they are able to estimate the utility patients derive from individual hospitals by observing the characteristics and choices of consumers who have access to the set of all hospitals (e.g., consumers who have enrolled in Medicare, Preferred Provider Organizations (PPOs), or indemnity plans). In many platform markets consumers with unrestricted choice sets do not exist, and the utility of individual affiliated products cannot be estimated without first accounting for the endogenous selection of consumers onto and across platforms.}

To estimate these effects, a selection problem must first be addressed. In networked industries, a consumer’s choice of platform is a function of not only the platform’s own characteristics, but primarily over the goods or services that are or will be available onboard. Different consumers make different platform choices based on their preferences over affiliated products: just as consumers choose a local community to best satisfy their preferences as in Tiebout (1956)’s model of local expenditures, so do they behave with respect to selecting a particular platform or hardware device.\footnote{Dubin and McFadden (1984) study a similar issue, in which household demand for electricity depends on the choice of space or water heating.} Since consumers who have purchased a platform are more pre-disposed to purchase those products onboard, failing to account for this selection will lead to significant upward biases in estimates of the quality and contribution of affiliated goods. In response to this challenge, I simultaneously estimate both hardware and software demand and control for the selection of consumers onto and across platforms using a nested fixed point routine.

This model also accounts for dynamic concerns such as the durability of goods and agents’ timing of purchases. This is critical in platform markets since the affiliation decision of consumers and firms often involves a long-term commitment; furthermore, a large literature has shown the limitations of applying a static methodology to a dynamic setting.\footnote{E.g., a consumer does not purchase a durable good that she has already purchased in the past, and upon purchase} To handle dynamics in hardware and software...
adoption and formulate and solve each consumer’s optimization problem, I adapt and extend a number of previously introduced techniques – including those pioneered in Rust (1987), Berry (1994), and Berry, Levinsohn, and Pakes (1995), and later synthesized in a dynamic environment by Hendel and Nevo (2006), Gowrisankaran and Rysman (2007) and Nair (2007). The extensions include the explicit handling of seasonality effects (crucial for most consumer product industries), the persistence of unobservable product characteristics (whose initial values may be endogenous), and the use of a more general evolution process for product qualities. For tractability, I assume consumers perceive the evolution of the expected lifetime utility from purchasing any product to follow a first-order Markov process, but allow beliefs over the future utility from purchasing a hardware platform to depend on its own current current value as well as those of its competitors and the time of year. Furthermore, consumer expectations are assumed to be rational in the sense they are consistent with actual realized empirical distributions of these values.

Using a new panel dataset containing monthly aggregate sales, prices, and characteristics for every hardware platform and software title in the sixth-generation of the videogame industry, I am able to identify consumer preferences for each hardware and software product through changes in purchase probabilities as software availability and product characteristics (e.g., prices) vary over time. The identifying assumption is that product unobservable characteristics are persistent, but innovations in these characteristics (i.e., monthly demand shocks) are uncorrelated with any changes in observable product characteristics. Furthermore, by observing the differential responses of affiliated and unaffiliated consumers (i.e., those who have/have not yet purchased a hardware platform) to the introduction of new software titles, I am able to identify heterogeneity in consumer preferences for software despite only observing aggregate market data. For example, in the absence of such heterogeneity, two different titles released at different points in time but purchased by the same share of consumers onboard a hardware platform should have the same impact on demand for the platform as a result of their introduction. However, if consumers are heterogeneous and early adopters of a platform are more pre-disposed to purchasing software than later adopters, then the title released earlier – despite selling to the same share of consumers onboard a platform – should have a different impact on demand for the platform than the title released later since the distribution of consumer preferences both on and off the platform will have changed.

Estimates from the demand system indicate that a platform’s gains from exclusive access to certain software titles can be large: although most titles do not have any substantial impact on hardware sales, certain hit titles may be able to increase the installed base of a hardware provider by 10% or more. In general, failure to account for dynamics, heterogeneity, and multiple purchases by consumers significantly biases the predicted impact a title has on the demand response of consumers.

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does not (usually) keep participating in the market; a consumer may purchase items in order to stockpile goods; and a consumer may have expectations over future product availability, pricing, and quality, and may delay purchase as a result. See e.g., Melnikov (2001), Song and Chintagunta (2003), Hendel and Nevo (2006), Carranza (2006), and Gowrisankaran and Rysman (2007) for other applications of modelling dynamic demand.
Hardware-Software Network Formation

The second part of the paper examines the contracting decisions between software firms and hardware platforms. When analyzing consumer demand, the set of software products onboard each platform at each point in time has been conditioned on and provided by the data. However, when institutional features of an industry change – as is the case when hardware platforms cannot vertically integrate into software provision or offer contracts contingent on exclusivity – it is unlikely that the contracting relationships between parties will remain the same.

To determine which platforms each software title joins, I define and compute a new equilibrium for a dynamic network formation game in which every title is allowed to freely choose which platforms to develop for, having observed the previous actions of consumers and other software titles. Capturing a key feature of any networked industry, the setup allows for the set of software products and consumers onboard each platform to change over time, with past actions influencing future decisions of market participants. I focus on an equilibrium in which each software title employs a strategy that depends only on the value and evolution of certain “payoff-relevant” state variables. Mirroring the assumptions used for the analysis of consumer demand, I restrict beliefs of all agents over the evolution of hardware and software mean-utilities to lie within the class of first-order Markov processes. Under this restriction, the solution concept used is equivalent to Markov Perfect Nash Equilibrium (Maskin and Tirole, 1988, 2001) and the model is similar in spirit to Ericson and Pakes (1995) and the literature that follows.

In order to compute a new equilibrium, I require an estimate of the profits each software title expects to receive if it develops for any set of platforms. The expected number of copies a given title would sell is obtained from the consumer demand estimates. However, construction of estimated profits also requires knowledge of underlying “porting” costs incurred by software titles in order to develop for different sets of platforms. These are unobserved. To estimate these costs, I assume each software title in the data not engaged in an exclusive arrangement chose to develop for the set of platforms which maximized its expected profit, and use an inequalities-based estimator developed in Pakes, Porter, Ho, and Ishii (2006) to recover an estimate of porting costs that best rationalize each title’s observed choice.

I next evaluate the fit of the estimated parameters and the dynamic network formation model by first fixing the decisions of titles observed to have been contractually exclusive in the data, and then computing an equilibrium which allows consumers and all other titles to freely choose which platforms to purchase or join. I find the estimates and model accurately predict platform installed base figures, market shares, and contracting decisions for “hit” titles.

Counterfactual Experiments

Finally, I analyze counterfactual regimes in the absence of exclusive arrangements between hardware platforms and software titles. To account for the possibility that eliminating exclusive arrangements

\[21\text{When platforms are forbidden to engage in exclusive deals, they are treated as passive agents. See Hagiu and Lee (2007) for a model of platform competition in which platforms actively bargain for exclusivity.}\]
may have affected the production of previously integrated titles, I compute two separate environments in which these titles are either assumed to still enter the market or are eliminated altogether. In both cases, simulations indicate that the industry is far less competitive when exclusive vertical arrangements are prohibited, with the incumbent capturing approximately 75% market share in hardware and nearly 90% of all software titles sold.

1.2 Road Map

In the next section I describe the U.S. videogame industry, the role of exclusive vertical arrangements, and important features and stylized facts which must be captured in any reasonable analysis. Section 3 overviews the theoretical issues involved with demand estimation in platform markets, and develops the full dynamic model with the accompanying details on estimation, inference, computation, and identification; estimation results are presented in section 4. I lay the groundwork for computing the dynamic network formation game in section 5, and also discuss how to recover the underlying porting costs borne by software firms. Finally, I analyze counterfactual regimes in which exclusive agreements are banned in section 6, and conclude in section 7.

2 Application: The U.S. Videogame Industry

Starting as a fringe industry in the early 1970’s with the introduction of a home version of *Pong*, the U.S. videogame industry has since grown to reach $13.5B in revenues in 2006. Increasingly, as evidenced by the widespread adoption of the new generation of consoles introduced in 2006, videogames have broadened their appeal and user base from a child’s hobby to something more mainstream: 69% of American heads of households engage in computer and videogames, with the average age of a player being 33 years old, and market penetration of videogame consoles reached 41% of U.S. television households (45M) in 2006.

A videogame system comprises a hardware platform (the “console”) and software (its games). In the current and most recent generations, each console is and has been provided by one firm – the platform provider – as a tightly integrated and standardized device which is required in order to run any of the titles provided for the system. Videogame software, on the other hand, is brought to market by two entities: *developers*, who undertake the programming and creative execution of each title; and *publishers*, who handle the marketing and distribution of a game. Publishers are usually integrated into software development and have their own in-house development studios; although independent software development studios do exist, as the costs of developing games have increased over time – average costs reached $6M during the late 1990’s – these studios still must

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22 Approximately $6.5B from console videogame software, $1B from PC videogame software, and the rest from console hardware and accessories. *Entertainment Software Association 2006 Sales, Demographic and Usage Data.*


25 NB: this is unlike the PC industry where a system’s hardware and operating system may be provided by different firms, and can be modified and configured further by the end user.
often turn to software publishers for financing in exchange for distribution and publishing rights.\textsuperscript{26}

Console manufacturers have also integrated into software publishing and development, with each platform provider typically having its own publishing unit and game development studios. Any title produced by the console maker via its own integrated studios or distributed by its own publisher is known as a \textit{first-party} title, and is exclusive to that hardware platform. All other games are \textit{third-party} titles and are published by other firms. Within a generation, games developed for one console are not compatible with other consoles; in order to be played on another console, the game must explicitly be “ported” by a software developer and another version of the game developed and published.\textsuperscript{27} These porting costs (which includes additional development and distribution costs) are non-negligible, and could range from a few hundred thousand to a few million dollars for supporting an additional console during the period analyzed in this paper.\textsuperscript{28} The choice of which platforms to develop for is highly strategic: a third-party software developer can release a title on multiple platforms in order to reach a larger audience but pay additional costs, or it can develop exclusively for one console and forego selling its game to consumers on other platforms. Even if a title chooses to be exclusive, it has multiple options: it can voluntarily be exclusive, enter into a publishing agreement with the console provider, or opt to sell the game or even entire studio outright.

Since videogame consoles usually have little if any stand-alone value, consumers typically purchase a particular console only if there are desirable software titles on that system.\textsuperscript{29} At the same time, software publishers release titles for consoles that either have or are expected to have a large installed base of users who potentially will purchase their games. These cross-side network externalities and “two-sidedness” are manifest in most hardware-software industries, and in particular yields a complex form of competition between rival platforms. On the pricing front most videogame console manufacturers subsidize the sale of their hardware devices to consumers, selling them close to or even below cost, and make profits by charging publishers and developers a royalty for every game sold on that platform.\textsuperscript{30}

As discussed in the introduction, as the dominant videogame platform provider during most of the 1980’s and 1990’s, Nintendo used to write forced exclusivity contracts with developers, committing them to 2-year exclusive deals in exchange for the right to develop for its system. This changed following a 1992 antitrust investigation related to \textit{Atari Games Corp v. Nintendo of America, Inc}, and Nintendo dropped these practices. Since then, such forced exclusivity contracts have not been observed. In their place, console manufacturers have primarily relied on internal development, integration, or favorable contracting terms to third party developers or publishers (e.g., lower royalty rates, lump sum payments, or marketing partnerships) in order to secure exclusive titles. More re-

\textsuperscript{26}Coughlan (2001).
\textsuperscript{27}A notable exception is “backwards compatibility,” which refers to the ability of a new console to use software developed for the previous version of that particular console. For example, each of Sony’s three home videogame consoles could play games made for the previous generation.
\textsuperscript{28}Industry sources; Eisenmann and Wong (2005) cite $1M as an estimate of the cost to develop a title for an additional console.
\textsuperscript{29}Recently however, consoles have been able to perform more tasks such as watch DVDs, access the internet, and purchase digital content or services.
\textsuperscript{30}See Hagiu (2006) and Evans, Hagiu, and Schmalensee (2006) for more on this point.
cently, as initial development costs for games have been increasing and porting costs have fallen as a percentage of total costs, most third-party titles have chosen to multihome in order to maximize the number of potential buyers; consequently, console providers have become more reliant on their own internal first-party titles to differentiate their platforms.

2.1 The Sixth Generation: 2000 - 2005

The videogame industry typically witnesses the release of a new set of consoles approximately every five years. Since hardware capabilities remain fixed within a generation to ensure compatibility and standardization, it is only during these generational shifts that new hardware with more powerful processing power and graphical abilities can be introduced. In October, 2000, Sony released its Playstation 2 (PS2) console, among the first of what has since been referred to as “sixth-generation” of videogame consoles.\(^{31}\) The PS2 was a followup to the original Playstation (PS1), Sony’s wildly successful entry in the previous generation.\(^{32}\) Sony also had the advantage of being the first out of the gate with its sixth-generation hardware console; only a year later did industry veteran Nintendo release its Gamecube (GC) console and new entrant Microsoft bring its Xbox console to market. By the time the first seventh-generation console was introduced in October 2005, Sony’s new console sold almost double the number of hardware devices as both its competitors combined.

This paper focuses on the sixth-generation for several reasons. First, it marked the arrival of a new competitor – Microsoft – to the industry. Itself a veteran and competitor in other platform industries, Microsoft’s entry tactics included the acquisition of several software developers; whether or not Microsoft would have been able to gain a foothold into the industry absent integration or exclusive contracting is an open question. Secondly, the three platform providers are the same as the current seventh-generation providers, providing timeliness to this line of inquiry. Finally, the sixth-generation marked the first steps towards placing the videogame industry firmly within the convergence battle between personal computers and other general consumer electronics. Starting with the introduction of DVD and online capabilities with these consoles, videogame platform providers have since added significant non-gaming functionality to their devices such that they now function as fully independent media hubs and platforms in several other markets.\(^ {33}\) Consequently, the success or failure of these hardware devices has a dramatic impact on industries far removed from only videogames.

2.2 Data and Descriptive Statistics

The main data comes from a new panel data set, and consists of monthly observations from January 1994 to October 2005. Each observation includes the average selling price and quantity sold of each

\(^{31}\)Sega’s Dreamcast, released a year earlier, was discontinued on January 31, 2001 and is not considered in this paper.

\(^{32}\)The PS1 had sold over 70M units by that point, and would go on to sell over 100M units by the end of 2005. ([http://www.playstation.com/business.html](http://www.playstation.com/business.html))

\(^{33}\)E.g., new videogame consoles are able to play next-gen HD-DVD/Blu-ray movie discs, download movies and digital content, watch TV over the internet, and network with other users.
videogame console, the average selling price, quantity sold, and other descriptive information for each software title on each console (including age appropriateness ratings provided by the industry *Entertainment Software Rating Board*, genre, and date of release). This data was obtained from the NPD Group, a market research firm.\(^{34}\) Prices are adjusted via the Consumer Price Index.\(^{35}\) In the data, there is information on a total of 12 consoles (two of which existed prior to 1994 and the other 10 of which were introduced during the period of the data), and 6606 videogame titles. However, since the analysis of this paper focuses on the sixth-generation, the data utilized is selected from the 61 month period between September 2000 and October 2005. During this period, three videogame consoles and 1581 unique software titles were released. The population of potential consumers is given by the number of television households collected on a yearly basis from Nielsen and linearly interpolated to the monthly level.

General descriptive statistics for each of the three sixth-generation consoles are provided in table 1. What follows are additional stylized facts about the industry:

- **Prices:** Hardware prices followed very steady paths, interrupted only by two major discrete jumps. The PS2 and Xbox started retailing for $300, but in May 2002 both simultaneously cut their prices by $100 prior to the “E3” industry trade show. Nintendo followed with a $50 price cut of its own from $200 to $150. Microsoft and Sony again quickly dropped prices one after the other two years later. Figure 1 illustrates the price paths of each hardware console. Software prices, however, follow much more regular price drops, with price cuts usually following the first few months of a new title’s release.

- **Seasonality:** The videogame industry, like most consumer product markets, exhibits considerable seasonality both in consumer demand as well as in software supply. Figure 1 also shows the number of total hardware consoles sold each month, and during holiday months (November and December) the number of consoles sold is easily double or triple the average number sold in other months. Software supply also exhibits significant variance across months: some have over 100 new titles released across systems, and others less than 5.

- **Exclusivity and Multihoming:** There is significant variation in the number of software titles that are exclusive across platforms: although nearly 64% of all unique software titles are exclusive to one console, the majority are located on the PS2. On the other hand, the majority of GC tiles are available on the other two console systems. At the same time, as seen in Table 2, the top titles for the GC are all exclusive whereas the PS2 has more nonexclusive titles in its top sellers.

- **Concentrated Software Sales:** Videogames, like motion pictures, are primarily a hit-driven industry where most sales are concentrated among a few top-selling games. Despite there being over 1500 unique titles released over the three sixth-generation consoles, the top 10

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\(^{34}\)The information is collected from approximately two dozen of the largest retailers in the U.S., which account for about 65% of videogame and console sales, and is extrapolated by NPD for the entire U.S. market.

\(^{35}\)All urban consumers, all items less food and energy.
listed in Table 2 on the PS2, Xbox, and GC accounted respectively for 18%, 33%, and 47% of total platform software sales. The concentration applies to software publishers as well – with over 150 publishers who have released a game for a sixth-generation console, the top 5 were responsible for 50% of all software sold (by total quantity sales), and the top 20 responsible for over 90%. Indeed, only 5 publishers command greater than 10% market share on any given console, and 3 of them are the console manufacturers themselves: Sony, Microsoft, and Nintendo. Finally, the concentration of software sales occurs early in a title’s life, with on average over 50% of total sales occurring within the first 3 months of release.

- Significant Consumer Heterogeneity: According to Nielsen, the heaviest 20% of videogame players account for nearly 75% of total videogame console usage (by hours played), averaging 345 minutes per day during the fourth quarter of 2006. At the same time, the fastest growing segment of users are known as “casual gamers” who spend less than 5 hours a week playing games.

3 Consumer Demand

In order to study the impact of a restriction on exclusive arrangements, an understanding of how consumer demand responds to changes in software availability is required. Not only is this used to estimate platform and software market shares following a change in industry structure, but it also is needed to understand the incentives governing each software title’s decision of which platform(s) to develop for in the first place. This section develops a structural model of consumer demand for both hardware and software in general platform-intermediated markets, and then applies it to estimate demand in the videogame industry.

I begin in subsection 3.1 by overviewing the theoretical issues involved with demand estimation in platform markets, and introduce the main structural innovations of the paper within a static environment. These include specifying platform utility as a function of affiliated software products and handling the selection of heterogeneous consumers across platforms. I discuss the shortcomings of previously used discrete choice methods, including why they are not sufficient to capture realistic substitution patterns or elasticities of demand within these markets, and why they are not able to recover the quality or impact on platform demand of an individual software title.

In subsection 3.2, I present the full dynamic model used in estimation which allows for consumer heterogeneity, multiple hardware system purchases, and forward-looking consumers. These consumers account for future software releases or price drops, and may delay purchase of a hardware platform or software title in anticipation of receiving higher utility in the future. The major assumption I use to model consumer expectations is similar to the one used in Melnikov (2001), Hendel and Nevo (2006), and Gowrisankaran and Rysman (2007): rather than explicitly model supply-side decisions (which includes software availability and pricing) when evaluating consumer expectations, I instead assume consumers use a reduced-form approximation (i.e., a first-order Markov process) for the evolution of each product’s expected lifetime utility of purchase. However,
I allow for more general beliefs in that consumers expect this future value for hardware to depend on its current value as well as those of other competing products and the time of year (in order to account for competitive and seasonality effects). These expectations are assumed to be rational in the sense that they are consistent with the realized empirical distribution of product qualities.

I also make the additional assumption that software titles are not substitutes, and each title effectively competes within its own market. Although a strong assumption (e.g., this effectively ignores consumer time and budget constraints), there is some evidence suggesting that any substitutability effects may not be first-order: for example, Nair (2007) finds empirically that videogames do not appear to be very substitutable for one another. These reduced form results are consistent with there being a large number of titles (even within a particular genre), each with their own distinct idiosyncrasies, plots, characters and style of play. Furthermore, even if substitution effects between certain titles exist, they may be less of an issue for “hit” titles; thus, relaxing this assumption should not dramatically change the main implications of the counterfactual exercises conducted later in this paper. The important implication this particular assumption delivers is that each consumer, after solving her appropriate dynamic policy for hardware purchase, can solve an independent optimal stopping problem for each individual piece of software.

In subsection 3.3, I develop the estimation and associated computational routine. Those not interested in details may skip ahead to subsection 3.4 where identification of the model is discussed, or to section 4 where results from the demand estimation are presented.

3.1 Demand Estimation in Platform Markets

Assume \( J \) denotes the set of hardware platforms and \( K \) the set of software products available in a given market. Let \( K_j \) represent the set of software available on platform \( j \). A consumer can only utilize a software product \( k \in K_j \) if she first purchases platform \( j \). For now, assume consumers may only purchase one hardware platform and the environment exists only for one period in order to abstract away from dynamic concerns. The timing of actions is as follows:

Stage I: Consumers may choose to purchase any hardware platform \( j \in J \).

Stage II: If consumer \( i \) has chosen platform \( j \), she may purchase any subset of products \( K_{i,j} \subset K_j \).

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36For software released between 1998-2000 on Sony’s original Playstation console, Nair (2007) shows cross-price effects across games to be low (even when accounting for strategic release timing on the part of game developers), consumers do not seem to exhibit intertemporal substitution within genres, entry by hit games do not have a significant effect on sales or prices of games within a genre, and rates at which game prices fall are independent of competitive conditions within the market.

37As will be evident, only the contracting decisions for high quality hit titles substantially affect platform market shares. Such titles have significant market power and may be the least likely to be affected by potential competition from other titles.

38Each software title \( k \) may be available on multiple platforms – i.e., \( K_j \cap K_{j'} \neq \emptyset \) need not be empty. As long as the econometrician can distinguish which platform a purchaser of software title \( k \) owns – which is the case in this paper – the following discussion does not change.
I focus on the problem of estimating a demand system for consumer behavior observed in stages I and II. I assume the econometrician observes the aggregate share of each hardware platform chosen in Stage I and share of consumers on each platform who buy each piece of software in Stage II. In this platform setting, there are two primary issues that a demand system must capture:

i. **Platform utility is a function of affiliated products $K_j$:** a consumer derives utility from purchasing a particular piece of software, and this must be accounted for in the utility she expects to derive from adoption of the platform. Any parameters that enter into the specification of utility of both software and hardware – e.g., price sensitivity – should be consistent and jointly estimated. Furthermore, a consumer’s utility upon joining a platform can be a function only of those products affiliated with that platform, and the choice set over software products changes depending on which platform she joins.

ii. **Consumers select across platforms according to their preferences and characteristics:** failing to account for heterogeneity in consumer preferences and their selection across platforms will lead to biased estimates of the quality and contribution of a piece of software to consumer utility, since those onboard the platform have already exhibited their preference for those goods affiliated. Consequently, any model which implies consumers who purchase and those who do not are identical is likely misspecified. Nonetheless, this is often the assumption made when estimating software demand without also explicitly accounting for hardware demand.\(^{39}\)

I have not yet specified when $K_j$ is determined. It may be the case that all software products join a platform prior to Stage I, all join immediately before Stage II but following Stage I, or some join before Stage I and others before Stage II. In the case where consumers perfectly observe the set of software products available on each platform $K_j$ and perfectly know their utility over each software product prior to platform adoption in Stage I, then there is no need to separate out the consumer’s decision into these two stages: consumers essentially choose “bundles” of both hardware and software simultaneously, and thus any discrete choice demand framework over properly specified bundles of goods would be adequate.\(^{40}\) Yet, in many environments consumers do not have complete information about either the identity of software products available, or the utility they derive from them.\(^{41}\) Insofar there is consumer uncertainty about software quality or availability prior to Stage

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\(^{39}\)To preview later discussion, this issue is amplified when demand is dynamic since there is an additional dimensionality of selection over time. In many hardware-software industries, early adopters tend to be those who exhibit high values for software or low price sensitivity. Without heterogeneity, the characteristics of outside consumers (non-purchasers) do not change over time, and any model would have difficulty rationalizing improving product quality and declining prices with falling shares of observed purchasers.

\(^{40}\)If there is uncertainty only over the utility derived from the platform itself – and not from software – then demand can be estimated in this manner as well. Such platform-only uncertainty may come from the consumer’s own idiosyncratic preferences or the actions of other consumers. E.g., consumers may realize certain needs subsequent to platform adoption or can derive utility from the adoption decisions of other consumers (network effects).

\(^{41}\)An example of this latter phenomenon would be that consumers do not know exactly the illness(es) that may beset them until after they join an HMO and become sick; consequently, since the utility from a hospital on an HMO plan is contingent on the type of illness acquired, consumers can only form expectations over the utility an HMO’s hospital network provides (Ho (2006)). In this setting, the uncertainty will be from the dynamic setting and the durability of the hardware platform: consumers do not know for certain what software titles will be available in the future.
I which is only resolved after the platform has been chosen, then there is a need to link software demand following the realization of uncertainty with the ex ante expected utility from software in hardware demand. For now, I assume $K_j$ is given and only perfectly known to consumers after they decide whether or not to purchase platform $j$.\textsuperscript{42}

Previous work on estimating demand in platform markets has usually taken one of two approaches: (i) estimate only one side of the market (typically the hardware side) using a reduced form approximation for the contribution of utility of the other side (which usually is the number of complementary products available);\textsuperscript{43} (ii) estimate each side in a separate two-stage procedure, combining software estimates obtained in the first stage to construct a measure for hardware utility estimated in the second stage.\textsuperscript{44} Although the first approach is adequate when consumers care only about product variety and the number of available products, it is not well suited to analyze markets where consumers care about the identity of firms on the other side. For example, a “killer-application” developed for a hardware system or world-class cancer treatment center as part of an HMO group are not the same as a handful of mediocre titles or mid-tier hospitals; as a result, platform utility needs to be a function of more than only the number of products available onboard. Failing to account for heterogeneity across firms and software titles precludes any hope of estimating their marginal contribution to a platform.

The second approach – a two-stage procedure – can yield reasonable results only if the econometrician observes all characteristics of consumers onboard each platform that influence their demand for software, or if there is a subset of consumers with unrestricted access to all software products whose preferences over software can separately be estimated and applied to those onboard a platform. Not only are the data requirements and restrictions more intensive, but this also rules out the possibility for controlling for selection on any unobservable characteristics. This problem is exacerbated once multiple periods are introduced, since a (static) two-stage estimation procedure also cannot consistently handle the dynamic evolution and selection of consumer heterogeneity across and onto platforms.

**Simultaneous Estimation**

The tight integration between hardware and software demand suggests moving towards a method which can simultaneously estimate both sides at once. I adopt a discrete choice based approach to demand estimation (see e.g. Lancaster (1971), McFadden (1973), Berry (1994) and Berry, Levinsohn, and Pakes (1995)).\textsuperscript{45} Let the total expected lifetime utility that a consumer derives from a single platform be given by

\[ U(\psi, x_j, \xi_j, \Gamma_j(\cdot); \theta) \]

\textsuperscript{42}Determining the set of software products onboard each platform is the focus of section 5.

\textsuperscript{43}See e.g. Nair, Chintagunta, and Dubé (2004) for PDAs; and Clements and Ohashi (2005), Prieger and Hu (2006), Corts and Lederman (2007), and Dubé, Hitsch, and Chintagunta (2007) for videogames.

\textsuperscript{44}See e.g. Town and Vistnes (2001), Capps, Dranove, and Satterthwaite (2003), and Ho (2006) for managed care organizations and hospitals.

\textsuperscript{45}These later models allow consumer preferences for product characteristics to vary as a function of observed and unobserved individual characteristics, and thus allow for more reasonable substitution patterns across goods.
where $\psi_i$ is a vector of individual characteristics and preferences, $x_j$ and $\xi_j$ are observable and unobservable product characteristics, and $\Gamma_j(\cdot)$ – which may also be a function of individual preferences and characteristics – represents the total expected utility a consumer derives from purchasing and using software available on platform $j$.\footnote{I am implicitly assuming total software utility enters linearly into a consumer’s calculation of hardware utility, so that the expected utility of software $\Gamma_j(\cdot)$ is a sufficient statistic for computing the expected lifetime utility from hardware.} As mentioned before, so that the choice of optimal bundle of software cannot be collapsed into the hardware purchase decision (which may be conceivable in a static environment, but not so in a dynamic environment studied later), I assume that there is some uncertainty over software quality or availability that is resolved only after the hardware is purchased. $\theta$ is a vector of parameters to be estimated, which includes any parameters governing the distribution of unobserved consumer characteristics and preferences.

A consumer purchases the platform that maximizes her utility, and chooses $j$ if and only if:

$$U(\psi_i, x_j, \xi_j, \Gamma_j(\cdot); \theta) \geq U(\psi_i, x_r, \xi_r, \Gamma_r(\cdot); \theta) \forall r \in J \cup \{0\}$$

where $j = \{0\}$ represents the “outside option” of non-purchase. Let

$$A_j = \{\psi : U(\psi_i, x_j, \xi_j, \Gamma_j(\cdot); \theta) \geq U(\psi_i, x_r, \xi_r, \Gamma_r(\cdot); \theta) \forall r \in J \cup \{0\}\}$$

denote the set of values for $\psi$ which induces consumers to choose good $j$. If $P_0(d\psi)$ denotes the (initial) population density of $\psi$, then the share of consumers who choose platform $j$ is given by

$$s_j(\{x_j, \xi_j, \Gamma_j(\cdot)\}_{j \in J}; \theta) = \int_{\psi \in A_j} P_0(d\psi)$$

With the exception of $\Gamma_j(\cdot)$, the hardware adoption model and aggregation is no different than previous discrete choice demand models. However, as noted earlier, $\Gamma_j(\cdot)$ is endogenous determined by the software side of the market.

To define $\Gamma_j(\cdot)$, it becomes necessary to examine the utility a consumer receives from each software title. Consider consumer who has purchased platform $j$, and now decides which subset of software titles to purchase. For expositional purposes, assume that each software product is in an independent market such that there are no substitution or complementarity effects across titles.\footnote{If there were such effects, the model can be extended by allowing a consumer to choose the optimal bundle of software titles over all possible bundles.} Thus, a consumer can decide whether or not to buy a particular title in isolation, and will purchase a given title if that title yields higher utility $U^sw$ than the outside good:

$$U^sw(\psi_i, w_k, \eta_k; \theta) \geq U^sw(\psi_i, w_0, \eta_0; \theta)$$

where now $w_k$ and $\eta_k$ are the observable and unobservable characteristics of title $k$. With independent software titles, the expected software utility on platform $j$ will simply be the maximum utility.
derived by purchasing or not purchasing each particular software title:

$$\Gamma_j(\cdot) = E[\sum_{k \in K_j} \max\{U^{sw}(\psi, w_k, \eta_k; \theta), U^{sw}(\psi, w_0, \eta_0; \theta)\}]$$

with the expectation over both the set of products $K_j$ as well as individual product utilities $U^{sw}(\cdot)$.

Although the set of consumer types $A_k$ who purchase good $k$ is defined similarly as on the hardware adoption side,

$$A_k = \{\psi : U^{sw}(\psi, w_k, \eta_k; \theta) \geq U^{sw}(\psi, w_0, \eta_0; \theta)\}$$

the share of consumers on platform $j$ who purchase $k$ is given by

$$s_k(w_k, \eta_k, w_0, \eta_0; s_j(\cdot), \theta) = \frac{\int_{\psi \in A_j \cap A_k} P_0(d\psi)}{s_j(\cdot)}$$

where now integral is over the intersection $A_j \cap A_k$. This is the selection issue discussed earlier: consumers who purchased platform $j$ are a selected subsample of the entire population, and it is only these consumers who must be considered when analyzing software demand.

For a given parameter vector $\theta$, most of the established estimation routines require the computation of shares $\{s_j(\cdot; \theta)\}_{j \in J}$ and $\{s_k(\cdot; \theta)\}_{\forall k \in K_j, \forall j}$: e.g., they are used in the likelihood function for MLE, or are required to recover unobservable characteristics $\xi$ and $\eta$ from which moments may be constructed (as in Berry, Levinsohn, and Pakes (1995)). However, without knowing the distribution of consumer types who select a given platform, calculating the share of consumers who purchase a particular software title is impossible; at the same time, computation of the distribution of consumer types who select a given platform requires knowledge of software quality, which itself is predicted from market shares derived from the software adoption side. More formally, for a given $\theta$, the share of consumers who purchase platform $j$ given by (1) cannot be computed without first knowing $\Gamma_j(\cdot)$; however, computation of $\Gamma_j$ will require calculation of the utility of each software title on platform $j$, which in turn requires knowledge of the distribution of consumers who have selected platform $j$ (i.e., the limits of integration in (2)).

This paper introduces a nested fixed point routine to control for this selection. I first fix the distribution of consumer types onboard each platform, and obtain a first-step estimate of the fraction of consumers who purchase each title. This allows for the construction of a first-step estimate of total software quality for each platform. I then update the distribution of consumer types onboard each platform using the new estimated software quality. This procedure is repeated, iterating between estimating hardware adoption (updating the distribution of consumer types) and software adoption (updating the quality of software onboard each platform) until convergence, which occurs when predicted values from the hardware side are consistent with predicted values on the software side. Further details are provided in subsection 3.3.
3.2 Dynamic Model of Consumer Demand

Although the previous discussion illustrated the importance of jointly estimating hardware and software demand and controlling for the selection of heterogeneous consumers across platforms, it did so in a static one-period environment. In many applications including the one examined here, a static environment is unrealistic. In the videogame industry, consumers internalize future software availability, quality differences, and potential price drops when deciding when and whether or not to purchase either a hardware system or software title. There is also a clear interdependence of demand across time: first, since videogame consoles and software are durable goods, consumers who have purchased a particular console or title in the past will not consider purchasing the same product in the future and the potential market size will shrink over time; furthermore, the types of consumers who purchase earlier will be different from those who purchase later: just as there is selection across platforms, there is also selection across time – e.g., early adopters of hardware platforms will tend to be less price sensitive or have a higher degree of affinity for software products.

Let $J_t$ denote the set of hardware consoles available for purchase at time $t$. Each consumer who has not yet purchased a console before may choose to do so, or wait until the next period. In addition, any consumer who has purchased a console by time $t$ may decide to purchase any software title $k \in K_{j,t}$ she has not yet already purchased, where $K_{j,t}$ represents the set of software titles available on platform $j$ at time $t$. Since hardware and software are durable goods and consumers have expectations over the evolution of their qualities and prices, the timing of purchase becomes a dynamic optimization problem.

I will first describe the consumer hardware adoption decision before discussing software adoption. For expositional purposes, I first introduce a model in which consumers may only purchase one hardware console before extending it to allow consumers to purchase multiple consoles.

**Hardware Adoption**

At any period, consumers who have not yet purchased a hardware platform may choose to do so. The total lifetime (expected) utility of consumer $i$ who purchases platform $j$ at time $t$ is given by:

$$u_{i,j,t} = \frac{\alpha^x x_{j,t} - \alpha^p p_i \ln(p_{j,t}) + \Gamma_{j,t}(\alpha^p, \alpha^\gamma) + \xi_{j,t} + \epsilon_{i,j,t}}{\delta_{i,j,t}}$$

where $x_{j,t}$ are observable characteristics of platform $j$ at time $t$ (which include a platform-specific and monthly fixed effects, age, age squared, and the current platform installed base), $p_{j,t}$ is the price of the console, $\Gamma_{j,t}$ is the expected present-discounted value of being able to purchase software for the platform in the current and future periods, $\xi_{j,t}$ is a product characteristic observable to the consumer but not to the econometrician, and $\epsilon_{i,j,t}$ is an individual-platform-time specific component which represents idiosyncratic consumer heterogeneity unobservable to the econometrician but realized by the consumer only at time $t$. Additionally, $\{\alpha^x, \alpha^p, \alpha^\gamma\}$ are (possibly individual specific) coefficients which reflect how intensely a consumer prefers platform characteristics, price, and software. The
actual functional form of $\Gamma_{j,t}(\cdot)$ emerges from the software adoption portion of the model which will be described in the next subsection; the only restriction made here is that $\Gamma_{j}(\cdot)$ differs across agents only as a function of their price sensitivity and software preference $\{\alpha_{p}^{i}, \alpha_{\gamma}^{i}\}$, and enters linearly into the utility specification. Finally, I denote the portion of a product’s individual specific utility net of the individual-specific unobservable by $\delta_{i,j,t}$; it can be thought of as the price-adjusted quality for platform $j$, and if $\epsilon_{i,j,t}$ were mean zero, it would represent the mean utility of such a purchase. Instead of purchasing a console, a consumer also may choose to wait and consume the outside good for one period – which yields utility $u_{i0t} = \epsilon_{i0t}$ – and return to the market in the next period.

In each period, a consumer chooses her optimal action – buy today or wait until next period – given her preferences, current product qualities, prices and software availability, and expectations over the evolution of these characteristics. A consumer’s utility from being on the market for a hardware platform – conditional on following her optimal policy – is given by the following value function:

$$V_{i}(\epsilon_{i,t}, \Omega_{i,t}) = \max \{ \max_{j \in J_{t}} u_{i,j,t}, u_{i,0,t} + \beta E[V_{i}(\epsilon_{i,t+1}, \Omega_{i,t+1}) | \Omega_{i,t}] \}$$  \hspace{1cm} (4)

where $\Omega_{i,t}$ includes current product attributes as well as the time of year (in order to account for seasonality effects) and any other market characteristics which may affect firm product pricing, entry, exit, or attributes. In general, it includes all variables at time $t$ in the consumer’s information set which affect her utility or value for waiting. This value function defines an optimal stopping problem which specifies when a consumer should (if ever) purchase a console.

For tractability, I impose the following assumption on the idiosyncratic utility shocks:

**Assumption 3.1.** $\{\epsilon_{i,t}\}_{t \geq i,t}$ are independently and identically distributed type 1 extreme value with variance normalized to $\pi^2/6$.

Whereas the normalization of the variance pins down the scale of utility (which is itself not identified), the distributional and conditional independence assumption allows (4) to be analytically integrated over $\epsilon$ and provide an “expected” value function ($EV$) for consumer $i$ as a function of current state variables:

$$EV_{i}(\Omega_{i,t}) = \int_{\epsilon_{i,t}} V_{i}(\epsilon_{i,t}, \Omega_{i,t}) d\epsilon = \ln(\exp\{\delta_{i,t}\} + \exp\{\beta E[EV_{i}(\Omega_{i,t+1}) | \Omega_{i,t}]\})$$  \hspace{1cm} (5)

where

$$\delta_{i,t} \equiv E_{\epsilon_{t}} \{ \max_{j \in J_{t}} u_{i,j,t} \} = \ln(\sum_{j \in J_{t}} \exp(\delta_{i,j,t}))$$

and represents the expected utility from purchasing a platform which delivers the maximal utility at time $t$.\(^{48}\) Also known as the “inclusive value” for consumer $i$, $\delta_{i,t}$ is a sufficient statistic for determining if the consumer will participate in the market at time $t$ by purchasing any platform.\(^{48}\)

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\(^{48}\)Ben-Akiva (1973) shows that $E_{\epsilon_{t}}[\max_{i} \{x_{i} + \epsilon_{i}\} | x_{i}] = \ln(\sum_{i} \exp(x_{i}))$ when $\{\epsilon_{i}\}$ are i.i.d. extreme value type 1. Application of this twice provides (5). See also Rust (1987) and Melnikov (2001) for similar applications.
Despite reframing the value function in expectations as in (5), the state space is still too large for the consumer’s optimization problem to be computationally solvable. One solution, utilized by Melnikov (2001), Hendel and Nevo (2006) and Gowrisankaran and Rysman (2007), is to assume that the inclusive values \( \delta_{i,t} \) within a market follow a first-order Markov process. However, although significantly reducing the computational burden of estimation, this assumption nonetheless imposes strong restrictions on the nature of industry competition and evolution.\(^{49}\) Rather than follow this approach, I instead assume that consumers perceive the mean utilities \( \delta_{i,j,t} \) for each console to evolve according to a first-order process which depends on previous values of itself in addition to \( \{\delta_{i,j',t}\}_{j' \neq j} \) of all other competing hardware platforms, as well as the time of year:

**Assumption 3.2.** Consumers perceive that \( \{\delta_{i,j,t}\}_{j \in J_t} \) can be summarized by a first-order Markov Process:

\[
F_i(\{\delta_{i,j,t+1}\}_{j \in J_{t+1}}|\Omega_{i,t}) = F_i(\{\delta_{i,j,t+1}\}_{j \in J_{t+1}}|\{\delta_{i,j,t}\}_{j \in J_t}, m(t)) \tag{6}
\]

where \( m(t) \) represents the month at time \( t \).

In addition to allowing each individual \( i \) to have different expectations over the evolution of the industry (\( F_i \) being individual specific), this specification is more general by allowing each product’s “quality” to evolve and as a function of – among other things – the proximity of other products’ quality to its own. Furthermore, which is crucial for this industry, any monthly seasonality is captured as a state variable. However, this assumption is still problematic in that it is difficult to create a supply model which generates first-order processes in \( \{\delta_{i,j,t}\} \).\(^{50}\) At the same time, as noted in Hendel and Nevo (2006), such first order processes may be reasonable approximations to consumer expectations and memory as consumers may remember prices and software availability from only their previous visit. Higher order processes not only may be too burdensome for estimation, but for consumer decision making as well.

Combining equation (5) with assumption 3.2, I can rewrite the consumer’s expected value function as:

\[
EV_i(\{\delta_{i,j,t}\}_{j \in J_t}, m(t)) = \ln(\exp(\delta_{i,t}) + \exp(\beta E[EV_i(\{\delta_{i,j,t+1}\}_{j \in J_{t+1}}, m(t + 1))|\{\delta_{i,j,t}\}_{j \in J_t}, m(t)])) \tag{7}
\]

where now the state space has been drastically reduced from \( |\Omega_{i,t}| \) to one of only \( \max_t \{\#J_t\} + 1 \) dimensions. Due to the limited number of platforms in the videogame industry (only 3 in the time period analyzed), this state space is now small enough for implementation.

\(^{49}\)E.g., the inclusive value for a consumer \( \delta_{i,t} \) may be high if there are few products with low prices or many products with high prices; a consumer would not only exhibit the same probability of purchasing but also have the same expectation over future values of \( \delta_{i,t} \) in both cases.

\(^{50}\)Setting aside dynamic issues due to the evolving nature of the installed and remaining customer base, there is a strong parallel to the industry dynamics model introduced in Pakes and McGuire (1994) and Ericson and Pakes (1995): if \( \delta_{i,j,t} \) represented platform \( j \)’s “index of efficiency” and platforms could engage in investment efforts to improve this value, then under certain assumptions (including the absence of other strategic control variables) there would exist a Markov-Perfect Nash equilibrium in which \( \{\delta_{i,j,t}\}_{j \in J_t} \) would evolve according to a first-order transition kernel.
Software Adoption

I now turn to analyze the software purchase decisions for a consumer which is used to construct the “software quality” function \( \{ \Gamma_{j,t}(\cdot) \}_{j \in J, t \in T} \) in (3).

In each period \( t \), a consumer who has either just or previously purchased platform \( j \) enters the market and may purchase any software title \( k \in K_{j,t} \) she has not yet purchased. Crucially, I assume each consumer views each software title as a separate market – i.e., the decision to purchase a title \( k \) is independent of purchasing \( k' \neq k \). Although this assumption is primarily motivated by feasibility,\(^{51}\) it may not be overly restrictive for reasons discussed earlier. For expositional purposes, the remainder of this subsection omits the \( j \) subscript for the platform and, unless otherwise specified, values are assumed to be platform specific.

A consumer’s expected lifetime utility from buying title \( k \) in period \( t \) (provided she already owns the platform) is given by:

\[
v_{i,k,t} = \tilde{\alpha}_i^\gamma + \tilde{\alpha}_w^w w_{k,t} + \tilde{\eta}_{k,t} - \alpha_i^p \ln(p_{k,t}) + \tilde{\epsilon}_{ikt} \tag{8}
\]

where \( w_{k,t} \) are observable software characteristics (which include a game-specific fixed effect, monthly fixed effects, as well as age, age squared, and the current installed base of previous purchasers), \( \tilde{\eta}_{k,t} \) is an software characteristic unobservable to the econometrician (but observable to the consumer), \( p_{k,t} \) the price, and \( \tilde{\epsilon}_{i,k,t} \) is an individual-software-time specific utility shock. \( \tilde{\alpha}_i^\gamma \) is an individual specific preference for “gaming” reflected in the increase in utility of any particular piece of software, and \( \alpha_i^p \) is the same coefficient used on the hardware side to capture a consumer’s price sensitivity. A consumer can also decide not to buy a piece of software at time \( t \) and return to the market in the next period, yielding the outside option utility \( v_{ik0t} = \tilde{\epsilon}_{ik0t} \).

Mirroring the hardware side, I make a distributional assumption on the individual-specific utility shocks:

**Assumption 3.3.** \( \{ \tilde{\epsilon}_{i,k,t}, \tilde{\epsilon}_{i,k0,t} \}_{\forall i,k,t} \) are independently and identically distributed extreme value type 1 with variance \( \sigma^2 \).

Here I allow for the variance of unobserved heterogeneity – \( \sigma_{\epsilon} \) – to vary between the hardware and software sides. I must account for this when combining measures of utility across sides as any shared coefficients (e.g., \( \alpha_i^p \)) need to be appropriately scaled.\(^{52}\) I thus re-express the software utility in (8) by multiplying and dividing through by \( \sigma_{\epsilon} \):

\[
v_{i,k,t} = \sigma_{\epsilon} \left( \alpha_i^\gamma + \alpha_i^w w_{k,t} + \eta_{k,t} - \alpha_i^{p,sw} \ln(p_{k,t}) + \epsilon_{i,k,t} \right) \tag{9}
\]

where \( \{ \alpha_i^\gamma, \alpha_i^w, \alpha_i^{p,sw}, \eta_{k,t}, \epsilon_{i,k,t} \} = \{ \tilde{\alpha}_i^\gamma, \tilde{\alpha}_w^w, \tilde{\alpha}_i^{p,sw}, \tilde{\eta}_{k,t}, \tilde{\epsilon}_{i,k,t} \} / \sigma_{\epsilon} \), and \( \zeta_{i,k,t} \) represents the (scaled) utility of purchasing a piece of software net of individual-specific-unobservable \( \epsilon_{i,k,t} \), and may also be

\(^{51}\)In a dynamic environment, individually tracking each consumer’s inventory and subsequent choice set is too computationally burdensome. Such problems were absent in the static environment used for exposition in 3.1.

\(^{52}\)C.f. Train (2003), ch. 2 for further discussion.
referred to as the price-adjusted quality or mean-utility for software $k$. To prevent confusion, I will use $\alpha_{i}^{p,\text{hw}} \equiv \alpha_{i}^{p}$ to refer to the coefficient on price used on the hardware side whenever appropriate.

A consumer’s optimal stopping problem for when (if ever) to purchase software title $k$ is given by:

$$W_{i}(\Omega_{i,t}, \epsilon_{i,k,t}) = \max \{v_{i,k,t}, v_{i,k,0,t} + \beta E[W_{i}(\Omega_{i,t+1}, \epsilon_{i,k,t+1})|\Omega_{i,t}]\}$$

Again, to reduce the dimensionality of the state space, the following assumption is made on the evolution of each software-title’s mean-utility:

**Assumption 3.4.** Consumers perceive that $\{\zeta_{i,k,t}\}_{\forall k}$ can be summarized by a first-order Markov process:

$$G(\zeta_{i,k,t+1}|\Omega_{i,t}) = G_{i,j}(\zeta_{i,k,t+1}|\zeta_{i,k,t}, m(t)) \tag{10}$$

where now $G_{i,j}$ is specific to individual $i$ and console $j$. It is, however, not title specific, and thus all software titles on a given platform are perceived by consumers to follow the same evolutionary path contingent on their price-adjusted quality and time of year. This assumption is subject to the same caveats and support as provided earlier for assumption 3.2.

Using assumptions 3.3 and 3.4, I rewrite the expected value function of being in market for software title $k$ on platform $j$ at time $t$ as:

$$EW_{i,j}(\zeta_{i,k,t}, m(t)) = \int_{\epsilon_{i,k,t}} W_{i,j}(\zeta_{i,k,t}, m(t), \epsilon_{i,k,t})dP_{\epsilon}$$

$$= \sigma_{i} \ln(\exp(\zeta_{i,k,t}) + \exp(\beta E[EW_{i,j}(\zeta_{i,k,t+1}, m(t+1))|\zeta_{i,k,t}, m(t)]))$$

which imbeds consumer $i$’s expectations over future prices and characteristics for software $k$.

To close the software adoption portion of the model, I need to link $\Gamma_{j,t}$ to the value of being on the market for software on platform $j$. This “total software utility” on platform $j$ can be separated into two parts: (i) the utility from software available in the present period, and (ii) the utility from new software that will arrive in future periods. Let this latter value be denoted $\Lambda_{j,t}(\alpha_{i}^{p}, \alpha_{i}^{\gamma})$, and let $K_{j,t}^{R}$ denote the set of software titles released on platform $j$ at time $t$. Then $\Gamma_{j,t}$ is given by:

$$\Gamma_{j,t}(\alpha_{i}^{p}, \alpha_{i}^{\gamma}) = \left[ \sum_{k \in \{\cup_{t'\leq t} K_{j,t'}^{R}\}} EW_{i,j}(\zeta_{i,k,t}, m(t)) \right] + \frac{\Lambda_{j,t}(\alpha_{i}^{p}, \alpha_{i}^{\gamma})}{(\text{Expected}) \text{ Future Software Utility}} \tag{11}$$

where the first term aggregates the expected utility of being on the market for each piece of software currently available; it imbeds the consumer’s optimal policy in its calculation.

Although all three consoles survived to the end of observed time period, other consoles in previous generations have prematurely “died” and left the market. To allow for this uncertainty, I allow consumers to believe a console will continue to have software released in the next period with probability $\beta_{\gamma}$, constant across time and consoles. Conditional on $\beta_{\gamma}$, if a consumer had perfect information over all future titles that would be released up to the terminal date $T$, future utility
would be given by

\[ \tilde{\Lambda}_{i,j,t} = \sum_{t'=1}^{T} (\beta_\gamma \times \beta)^{t'} \left( \sum_{k \in \mathbf{K}_j^R} EW_{i,j}(\zeta_{i,k,t+t'}, m(t + t')) \right) \tag{12} \]

However, a consumer does not know exactly the number nor quality of future titles. I deal with this by assuming consumers have rational expectations consistent with the observed data, and condition on current observed market variables and product characteristics; i.e., \( \Lambda_{i,j,t} = E[\tilde{\Lambda}_{i,j,t}|\Omega_{i,t}, \beta_\gamma] \). In estimation, I use a nonparametric series regression of (12) on console characteristics and software availability to form an approximation of consumers’ expectations.

In the appendix, I discuss an alternative formulation for future software utility in which consumers believe the number and quality of titles released in each period are drawn independently from estimated distributions consistent with the data.

**Multiple Hardware Purchases**

When consumers are allowed to multihome and purchase multiple consoles, both the dynamic optimization problem and the expected lifetime utility from hardware purchase changes. First, a consumer upon purchasing a console no longer leaves the market, but retains the option value of returning in a future period and acquiring a second or even third console. Second, the utility derived from a console will be different if it is the first, second, or third purchased: the expected utility derived from purchasing an additional console should not include utility from a software title the user already had access to on her original console; there may also exist complementarity or substitution effects between multiple consoles.

To account for multiple hardware purchases, I introduce a new state variable indicating consumer \( i \)'s inventory of hardware consoles she already owns at time \( t \), and represent this by \( \iota_{i,t} \in \mathbf{I} \equiv \{0, 1\}^3 \). The expected lifetime utility from purchasing a new console now becomes a function of which consoles a consumer already owns, and is given by:

\[
u_{i,j,t}(\iota_{i,t}) = \alpha_i^x x_{j,t} - \alpha_i^P \ln(p_{j,t}) + \Gamma_{j,t}(\alpha_i^P, \alpha_i^7; \iota_{i,t}) + D(\iota_{i,t}) + \xi_{j,t} + \epsilon_{i,j,t} \tag{13}\]

where software utility \( \Gamma_{j,t} \) is now a function of \( \iota_{i,t} \) and is adjusted to account for the fact that a user may have already had access to certain titles on the consoles she already owns: i.e., \( \Gamma_{j,t}(\cdot; \iota_{i,t}) \) is defined as in (11), except now only includes those titles \( k \in \mathbf{K}_j \setminus \{\mathbf{K}_j \cap \{\mathbf{K}_{j'}\}_{j' \in \iota_{i,t}}\} \).

I also introduce a new term \( D(\iota_{i,t}) \) which denotes any complementarity or substitutability effects that may exist with ownership of multiple consoles.\(^{53}\) Conceptually this term should be negative, as the utility of a console should decrease if another is already owned. Since most of the diminished utility is from eliminating the double counting of non-exclusive software titles already

\(^{53}\text{This is similar to the additive complementarity or substitution term used in Gentzkow (2007).}\)
accessible on a console already owned, any remaining effects should come only from other factors such as time constraints or diminishing returns from gaming. In estimation, I assume $D(\cdot) = D$ if a consumer already owns at least one other console, and $D(\cdot) = 0$ otherwise. Although possible to be more general, estimation of a single constant is sufficient to capture the possibility that a previously purchased console reduces the future utility from another device beyond that of reducing the number of new titles that can be accessed.

Finally, I modify assumption 3.2 as follows:

**Assumption 3.5.** For each inventory state $i \in I$, consumers perceive that $\{\delta_{i,j,t}(i)\}_{j \in J_i}$ can be summarized by a first-order Markov Process:

$$F(\{\delta_{i,j,t+1}(i)\}_{j \in J_{i+1}} | \Omega_{i,t}) = F_i(\{\delta_{i,j,t+1}(i)\}_{j \in J_{i+1}} | \{\delta_{i,j,t}(i)\}_{j \in J_i}, i, m(t))$$

where now the evolution is not only individual specific, but inventory state specific as well.

In the appendix, I discuss further details required to integrate multihoming into the model.

### 3.3 Estimation and Computation

Berry, Levinsohn, and Pakes (1995) and following literature typically estimates discrete choice models by recovering the set of unobserved product characteristics for any parameter vector $\theta$ which perfectly rationalize the model’s predicted market shares with observed market shares, and then using a generalized methods of moments (GMM) estimator based on forming conditional moments with these unobserved characteristics: i.e., the identifying condition is typically $E[\xi | z] = 0$, where $z$ is a set of instruments orthogonal to $\xi$. However, this process requires finding appropriate instruments for product characteristics including price, which may not exist in the data.

Rather than proceed in this fashion, I instead leverage the dynamic aspect of my data and estimate based on the predicted evolution of the unobserved product characteristics. Namely, I assume that the unobservable characteristics for each hardware system and software title evolve according to an exogenous Markov process where these changes are independent from changes in observed characteristics. In turn, I estimate via (conditional) maximum likelihood (ML) on the implied changes in product unobservables.

Formally, the econometrician observes the characteristics and quantities sold for each console and software title in every period. Let $r_j$ denote the release date for console $j$ and $r_k$ the release date for software title $k$. From the model, the implied values of unobserved product characteristics

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54 Other applications include Nevo (2001) and Petrin (2002).

55 Berry, Levinsohn, and Pakes (1995) (in section 4) notes the possibility of proceeding in this fashion when utilizing panel data. In simultaneous work, Sweeting (2007) uses a similar assumption on the evolution of unobserved characteristics within a GMM estimator; as noted therein, this assumption is similar to the timing assumption used in the literature on the structural estimation of production functions to address the endogeneity of input choices (e.g., Olley and Pakes (1996), Levinsohn and Petrin (2003), Ackerberg, Caves, and Frazer (2006)). Here, although certain observable product characteristics are endogenously determined – such as pricing, the release of new software products, and changes in the installed base of each console – I assume their next period values are determined prior to the realization of changes in unobservable product characteristics.
\{\xi_{j,t}\}_{j \in J, t \leq t} and \{\eta_{j,k,t}\}_{j \in J, k \in K, t \leq t} can be computed as a function of parameters \( \theta \) in order to rationalize observed market shares with predicted market shares. I assume the following:

**Assumption 3.6.** Unobserved product characteristics for each console and software title evolve according to a first-order autoregressive (AR(1)) process, where the errors

\[
\begin{align*}
\nu^{hw}_{j,t}(\theta) &= \xi_{j,t}(\theta) - \rho^{hw} \xi_{j,t-1}(\theta) \quad \forall j \in J_{t-1} \\
\nu^{sw}_{j,k,t}(\theta) &= \eta_{j,k,t}(\theta) - \rho^{sw} \eta_{j,k,t-1}(\theta) \quad \forall j \in J_{t-1}, \forall k \in K_{j,t-1}
\end{align*}
\]

are independent of each other and changes in all observed characteristics, and are identically distributed according to probability densities \( f^{hw}(\cdot; \theta) \) and \( f^{sw}(\cdot; \theta) \).

The likelihood of observing a set of these errors is given by:

\[
L(\theta) = \prod_{j=1}^{\#J} \left( \prod_{t=r_{j}+1}^{T} f^{hw}(\nu^{hw}_{j,t}(\theta); \theta) \times \prod_{t=r_{k}+1}^{T} \prod_{k=1}^{\#K_{j,t}} f^{sw}(\nu^{sw}_{j,k,t}(\theta); \theta) \right)
\]

and the log-likelihood by:

\[
\mathcal{L}(\theta) = \sum_{j=1}^{\#J} \left( \sum_{t=r_{j}+1}^{T} \ln f^{hw}(\nu^{hw}_{j,t}(\theta); \theta) + \sum_{t=r_{k}+1}^{T} \sum_{k=1}^{\#K_{j,t}} \ln f^{sw}(\nu^{sw}_{j,k,t}(\theta); \theta) \right)
\]

The estimate of \( \theta_0 \) is the value of \( \theta \) that maximizes (16):

\[
\hat{\theta} = \sup_{\theta \in \Theta} \mathcal{L}(\theta)
\]

The initial values of \( \{\xi_{j,r}\}_{j \in J} \) and \( \{\eta_{j,k,r}\}_{j \in J, k \in K, t} \) are conditioned on in the specification of the likelihood. However, as \( T \) grows large, the contribution of these initial values to the likelihood grow negligible. Provided \( |\rho| < 1 \), the exact ML estimator and this conditional ML estimator which omits the first \( \xi_{j,r} \) and \( \eta_{j,k,r} \) term will have the same asymptotic distribution. Leveraging the dynamic aspects of the problem in this fashion not only handles the initial conditions problem, but also proves robust to the possibility that hardware and software release dates are “timed”: e.g., titles that have a relatively high initial unobserved quality \( \eta_{j,k,r} \) may systematically be released during “high” demand months, such as the beginning of summer and during the holiday season. As long as the evolution of unobservable characteristics continues to follow (15), this strategic timing will not bias estimates.

The following proposition proves this estimator is consistent and asymptotically normal.

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56 The stationarity coefficients \( (\rho^{hw}, \rho^{sw}) \) need to be estimated as well. The drifts of these processes are fixed to be 0 since they are not separately identified from the level of product fixed effects contained within \( \alpha^s \) and \( \alpha^w \).

57 See e.g. Fuller (1996); the estimator is also similar to the “\( y_0 \)-conditional estimator” described in section 8.7 of Hayashi (2000).

58 I.e., the values of \( \xi_{j,r} \) or \( \eta_{j,k,r} \) at release may be correlated with observable product characteristics.

59 See Einav (2007) for issues regarding seasonality and timing in the U.S. Motion Picture Industry.
Proposition 3.7. Let $n_{j}^{hw} = \sum_{t=r_{j}}^{T} 1$ and $n_{j}^{sw} = \sum_{t=r_{k}}^{T} \sum_{k=1}^{K_{j,t}} 1$ be the number of $\nu^{hw}$ and $\nu^{sw}$ observations for platform $j$, and $N = \sum_{j=1}^{J} (n_{j}^{hw} + n_{j}^{sw})$ be the total number of observations across platforms. Provided certain identifying and invertibility assumptions are satisfied (see assumption C.1 in Appendix), if $\hat{\theta}$ is a solution to (17), then as $T \to \infty$, $\mu_{j}^{hw} = n_{j}^{hw}/N$ and $\mu_{j}^{sw} = n_{j}^{sw}/N$ constant.

I. $\hat{\theta} \to_{p} \theta_{0}$

II. $\sqrt{N}(\hat{\theta} - \theta_{0}) \to_{d} N(0, J_{0}^{-1})$, where

$$J_{0} = \left[ \sum_{j=1}^{J} \mu_{j}^{hw} E\left( \frac{\partial \ln f^{hw}}{\partial \theta_{r}} \frac{\partial \ln f^{hw}}{\partial \theta_{s}} \right) \bigg|_{\theta=\theta_{0}} + \mu_{j}^{sw} E\left( \frac{\partial \ln f^{sw}}{\partial \theta_{r}} \frac{\partial \ln f^{sw}}{\partial \theta_{s}} \right) \bigg|_{\theta=\theta_{0}} \right]$$

(18)

Proof. See Appendix.

The proof requires modifying the standard asymptotic arguments for maximum likelihood estimators (e.g., Rao (1973)) by accounting for the non-identical distribution of the hardware and software errors in addition to the varying number of observations across platforms and time. For estimation, I will assume that the errors are normally distributed with $\nu_{j,t}^{hw} \sim N(0, \sigma_{hw}^{2})$ and $\nu_{j,k,t}^{sw} \sim N(0, \sigma_{sw}^{2})$.

Note also $\xi$ and $\eta$ cannot be computed exactly but rather only approximately due to both population sampling error and the need to simulate the integrals defining market shares for each product.\(^{60}\) As in Berry, Levinsohn, and Pakes (1995), I assume the population sampling is negligible due to the large sample size of over 100M U.S. households. However, the issues introduced with the need for simulation cannot be ignored. As has well been documented, simulation error is problematic in contexts where market shares are small (Berry, Linton, and Pakes (2004)), and also introduces a bias due to the non-linearity of the log transformation in the maximum likelihood estimator (Lee (1995)). To avoid problems involved with simulation error, I instead discretize the distributions of consumer heterogeneity and assume consumers belong to one of several “types” of consumers with identical $\{\alpha_{i}, 0, \beta \}$ coefficients (but with different realizations of $\{\epsilon_{it}\}$).

To calculate standard errors, I use the estimate $\hat{\theta}$ and take the sample analogue of (18).

Parameters to Estimate

Let $\theta_{1} = \{\rho^{hw}, \rho^{sw}, \alpha_{0}^{p,sw}, \sigma_{\alpha^{p},sw}, \sigma_{\alpha^{s},sw}, \sigma_{\epsilon}, \beta, D\}$ and $\theta_{2} = \{\alpha^{x}, \alpha^{w}\}$. The parameters to be estimated are $\theta = \{\theta_{1}, \theta_{2}\}$.

Computation

Here I overview the procedure of recovering the unobserved product characteristics $\{\xi_{j,t}(\cdot)\}_{j \in J_{t}, t}$ and $\eta_{j,k,t}(\cdot)_{j \in J_{t}, k \in K_{j,t}, t}$ as a function of the parameter vector $\theta$. Once these values are obtained,
the log-likelihood function in (16) can be computed.

The approach for recovering the unobservable utility components $\xi$ in the hardware side and $\eta$ in the software side is similar to the approaches utilized in Nair (2007) and Gowrisankaran and Rysman (2007), which both in turn nest the methodologies of Rust (1987), Berry (1994), and Berry, Levinsohn, and Pakes (1995). This paper, however, links the hardware and software demand systems and estimates both sides simultaneously; to do so, I introduce and employ a new nested fixed point routine in order to control for the selection of heterogeneous consumers across platforms and time. Additionally, this paper allows each platform’s mean utility to evolve separately from the inclusive value of the industry and explicitly accounts for seasonality effects.

For a given parameter vector $\theta$, I first obtain starting values for $\{\Gamma_{j,t}\}_{j \in J_t, \forall t}$ which can either be fixed (e.g., at 0) or can be computed by assuming that the distribution of consumer heterogeneity across new console purchasers is stationary and first estimating the software side. I employ the latter approach. Utilizing these initial values for $\{\Gamma_{0,j,t}\}_{j \in J_t, \forall t}$, I first estimate the hardware adoption side. The “mean” platform utilities $\{\delta_{j,t}\}_{j \in J_t, \forall t}$ which rationalize predicted market shares to observed market shares is found via a contraction mapping introduced in Berry, Levinsohn, and Pakes (1995). For each iteration of this contraction mapping, each consumer’s belief process over the evolution of $\delta_{i,j,t}$ for every inventory state $\iota$ is updated according to the regression

$$F_i(\delta_{i,j,t+1}(t)|\{\delta_{i,j,t}(t)\}_{j \in J_t, t, m(t)}) = \varphi_{i,j,t,0} + \sum_{j' = 1}^{3} \varphi_{i,j',t,1}\delta_{i,j',t}(t) + \sum_{m = 1}^{11} \varphi_{i,j,t,m+3}\chi_{m}(t) + \upsilon_{i,j,t,t}$$

(where $\chi_{m}(t)$ are indicator variables if $t$ is in month $m$) and her optimal stopping problem is solved to determine the probability of purchase.61 At each point in time, the number and identity of consumers at each inventory state evolves according to how many consumers of each type are predicted to have purchased a console (a process given by (32) in the Appendix), which at the end is aggregated across consumers to form predicted market shares for each month.

Once the hardware adoption side is computed for values $\{\Gamma_{n,j,t}\}_{j \in J_t, \forall t}$ (where $n$ denotes the iteration of the procedure), I use the probability that each consumer adopts a hardware platform in each period to form the consumer distribution of each hardware’s installed base across time, denoted by $\{dP_{j,t}(\alpha^p, \alpha^\gamma)\}_{j \in J_t, \forall t}$. This updated distribution of consumer types is then used to estimate the software adoption decision, which proceeds via a similar nested framework as the hardware side except that it now must be done for each console separately. I.e., the same contraction mapping is used to recover mean utilities $\{\zeta_{j,k,t}\}_{j \in J_t, k \in K_{j,t}, \forall t}$ for each piece of software on a given console,

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61 Unlike using more lagged terms, which would increase the state space and be too computationally expensive, the functional form can be expanded to utilize higher order terms and/or interactions between $\delta_{j,t}$ and its competitors $\{\delta_{j',t}\}_{j' \neq j}$. 

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where in each iteration consumer expectations are updated according to

\[ G_{i,j}(\zeta_{i,k,t+1}|\zeta_{i,k,t}) = \varphi_{i,0}^j + \varphi_{i,1}^j \zeta_{i,k,t} + \varphi_{i,2}^j \zeta_{i,k,t}^2 + \sum_{m=1}^{11} \varphi_{i,m+2}^j \chi_m(t) + \nu_{i,k,t}^j \]  

(20)

and the consumer’s optimal stopping problem is solved. After \( \{\zeta_{j,k,t}\}_{j \in J_t, k \in K_{j,t}, v_t} \) converge for a given set of probability distributions \( \{dP_{j,t}^m(\alpha^p, \alpha^s)\}_{j \in J_t, v_t} \), updated values of \( \{\Gamma_{j,t}^n\}_{j \in J_t, v_t} \) are computed for every inventory state. Future software utility in (11) is obtained via a non-parametric series regression on (12) using third-order terms and a full set of interactions on a console’s age, number of active software titles, and month dummies.

Finally, updated values \( \{\Gamma_{j,t}^n\}_{j \in J_t, v_t} \) are fed back into the hardware adoption side, which is then re-estimated. The procedure iterates between estimating the hardware and software sides until \( \{\Gamma_{j,t}\}_{j \in J_t, v_t} \) and \( \{dP_{j,t}(\alpha^p, \alpha^s)\}_{j \in J_t, v_t} \) converge, at which point \( \xi \) and \( \eta \) can be recovered from the final computed values \( \delta_{j,t} \) and \( \zeta_{j,k,t} \) via a linear regression. A non-derivative based Nelder and Mead (1965) simplex algorithm is used to search for \( \theta_1 \).

The procedure is illustrated in figure 2 with further details provided in Appendix A.3. With an appropriately fine grid and large enough bounds on the state spaces, no problems with convergence for \( \delta_{j,t}, \zeta_{j,k,t}, EV_i, EW_i \), or \( \Gamma_{j,t} \) were encountered.

### 3.4 Identification

As is often the case with estimating dynamic processes, without providing restrictions on the discount factor and parameterizing consumer heterogeneity, the model remains unidentified (Rust (1994), Magnac and Thesmar (2002)). I fix the monthly discount rate \( \beta \) at .99. I assume price sensitivity for hardware and software takes the form \( \alpha^{pl}_i = \alpha^{pl}_0 - \sigma^{pl}_i y_i \) for \( l \in \{hw, sw\} \), where \( y_i \) is consumer \( i \)'s annual household income, and \( \alpha^{pl}_0 \) and \( \sigma^{pl}_i \) are parameters to be estimated. As in Berry, Levinsohn, and Pakes (1995), I assume disposable household income \( y_i \) for the population is (independently) distributed log normally with mean and standard deviation estimated separately from the March 2001 Current Population Survey (CPS), and I draw from this distribution. Finally, I assume consumer preferences for software \( \alpha^s \) is independently distributed normally with standard deviation \( \sigma_s \); since \( \alpha^s \) enters linearly in utility, its mean is not separately identified from shifts in each software title’s fixed effect and is normalized to 0.

Even with these restrictions, there still remains the question of what precisely identifies the elements of \( \theta \). Unlike in static environments, the dynamic panel nature of the data used allows for repeated observations of hardware and software characteristics. Consequently, the components

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62 As shown in the appendix, the non-linear search is only over \( \theta_1 \), since \( \theta_2 \) can be “concentrated” out as a function of \( \theta_1 \) – i.e., the likelihood can be expressed as \( L(\theta_1, \theta_2(\theta_1)) \).

63 This data reports the distribution of disposable household income in 2000, which corresponds to the start of the data. From the Census Bureau, disposable income is defined as including money income, the value of noncash transfers (food stamps, public or subsidized housing, and free or reduced-price school lunches), imputed realized capital gains and losses, and imputed rate of return on home equity. It deducts imputed work expenses, federal payroll taxes, federal and state income taxes, and property taxes on owner-occupied homes.
of $\alpha^x$ and $\alpha^w$ – which include product and month level fixed effects as well as age, age squared, and installed base terms – are identified from the time variation in sales as these characteristics change.\footnote{Although not necessary, assuming age effects are shared across hardware platforms and across software titles further helps aid in identification. I assume monthly seasonality effects are the same across years and shared across hardware platforms, but may differ across platforms for software demand.} In a similar fashion, $\sigma_\epsilon$ (the ratio between software utility and hardware utility, which affects how $\Gamma_{j,t}$ enters into hardware demand) and $\beta_\gamma$ (the additional discounting assigned to future games that have not yet been released) are identified from changes in hardware demand in response to the release of new software titles.

The identification of the mean and variance of the consumer heterogeneity parameters and complementarity factor $D$ requires greater discussion, especially in the absence of micro-level data. First, mean household price sensitivities $\{\alpha_{0,l}^{p,l}\}_{l \in \{\text{hw,sw}\}}$ are identified via monthly variation in prices. Since there is very little variation in hardware prices over time (only 2 significant price drops for each console) but far more in software prices, I estimate $\alpha_{0,sw}^{p,sw}$ and leverage the restriction imposed by the model that $\alpha_{0,sw}^{p,hw} = \sigma_\epsilon \alpha_{0,sw}^{p,sw}$. I find that estimating $\alpha_{0,sw}^{p,hw}$ and $\alpha_{0,sw}^{p,sw}$ separately cannot reject this restriction, and thus only report estimates from the restricted specification.\footnote{I also scale the variance of hardware price sensitivity in a similar manner: i.e., $\sigma_{\alpha_p,\text{hw}} = \sigma_\epsilon \sigma_{\alpha_p,sw}$.}

Typically, the variance in consumer preferences $\{\sigma_{\alpha_p,sw}, \sigma_{\alpha_\gamma}\}$ can be identified from variation in product characteristics: as characteristics change for one product, substitution to products with similar characteristics indicate the presence of heterogeneity; on the other hand, if consumers substitute equally to and from all goods, then consumers are likely to be more homogeneous in their preferences. Unfortunately, this argument has limited power when there are only 3 hardware platforms in the data, and substitution away from a particular software title is limited to only the outside good (i.e., not purchasing the title and waiting until the next period).

Nonetheless, the dynamic nature of the panel data provides another means of identification for consumer heterogeneity: the endogenous shift in the distribution of consumer valuations over time. In this model, consumers who are less price sensitive or have a higher intensity for gaming will select out and adopt a platform earlier than others, creating a difference in the composition of non-adopters and adopters of a hardware system. If household heterogeneity in either $\alpha^p$ and $\alpha^\gamma$ is substantial, then consumer responses over time to changes in price or software availability on a given platform versus for a given platform will be different. For example, in the absence of heterogeneity in $\alpha^\gamma$, two different titles released at different points in time but purchased by the same share of consumers onboard a platform should have the same impact on demand for that platform as a result of their introduction: if there was no selection across time and consumers were relatively homogeneous, early adopters have the same preferences as later adopters, and the predicted quality of both games will be the same. However, in the presence of heterogeneity, the installed base of a console will have a higher share of consumers with a high value of $\alpha^\gamma$ earlier than later. As such, a title released later in a console’s history that attracts the same share of consumers as a title released earlier is actually a “higher” quality title (as reflected by a higher title-fixed effect), since being released later means it must have appealed to a less predisposed base of users;
consequently, it will have a different impact on demand for the platform than the other title. Thus, observing consumer demand for both a software title and for the platform as a result of that title’s introduction allows for the identification of consumer heterogeneity in gaming preferences ($\sigma_{\alpha\gamma}$). Similarly, heterogeneity in price sensitivity is identified from observing how the price sensitivity of consumers onboard a platform changes over time.

Finally, I discuss the identification of the complementarity term $D$. Since I fix the total market size for video game consoles and assume it to be equal to the number of television households, identification of $D$ comes immediately from its impact on the rate of change in the remaining market size. If $D$ is extremely high, then any purchaser of a hardware system essentially “leaves” the market and does not purchase another console; when $D$ is low, previous purchase does not remove that consumer from consideration when purchasing other hardware devices. As the potential market size directly enters into calculation of the observed and expected share of consumers who purchase products, the rate at which these shares change (and its implied impact on predicted product unobservables) identifies $D$. It is important to stress that even with consumer heterogeneity, the intuition remains the same. Aiding identification is the staggered introduction of consoles in the dataset: the PS2 was released a year earlier than the other two systems. Thus, whereas the change in sensitivity to price or software availability on the PS2 in the first year identifies the degree of consumer heterogeneity, whether or not these same early adopters are still “on the market” to subsequently purchase the Xbox or GC is a function of $D$.

4 Demand Estimation Results

Parameter estimates from the demand system are presented in Table 3. Multiple specifications are provided: column (i) estimates a static model without consumer heterogeneity (i.e., a standard logit model); (ii) and (iii) introduces dynamics without heterogeneity, but differ in whether or not consumers can singlehome or multihome; and finally, (iv) and (v) introduces consumer heterogeneity and dynamics when, again, consumers may singlehome or multihome. Estimation of models (i)–(iii) without any consumer heterogeneity is equivalent to estimating the hardware and software side sequentially in two separate stages; the nested fixed point routine introduced in the previous section to handle the selection of consumer heterogeneity is unnecessary. I will first describe demand results under the full model (v) before comparing across specifications.

Nonlinear Parameter Estimates

All non-linear parameters $\theta_1 = \{\rho^{hw}, \rho^{sw}, \alpha_0^{p,sw}, \sigma_{\alpha p,sw}, \sigma_{\alpha \gamma}, \sigma_\xi, \beta_\gamma, D\}$ except complementarity factor $D$ are estimated to be significant, including both parameters governing the variance in consumer heterogeneity. Signs of coefficients are as expected, with utility decreasing from price and having purchased a previous console. Regarding heterogeneity in price sensitivity, recall $\sigma_{\alpha p,sw}$ is the coefficient on a consumer’s annual household income and not the standard deviation of the distribution (which explains its small magnitude).
Heterogeneity in $\alpha^\gamma$ – a consumer’s taste for software and gaming – is substantial. The estimated value of $\sigma^\gamma = .77$ indicates that a consumer at the 80% percentile of the distribution sees a game as 4 times cheaper than a consumer at the 20% percentile of $\alpha^\gamma$ – i.e., a game selling at $50$ for the intense gamer is seen as costing $200$, holding all else equal, for the less interested gamer. Thus, it is unsurprising that most consumers at the lower end of the distribution of $\alpha^\gamma$ do not purchase a console let alone many games. Figure 3 illustrates the estimated composition of the installed base of consumers across console by quintile of the $\alpha^\gamma$ distribution. For the first two years of each console’s existence, over half of the users are in the top quintile of the distribution of $\alpha^\gamma$; it is not until the end of the console’s life-cycle that consumers with lower valuations begin to make up a significant share of users.

The ratio of the scale of the individual specific idiosyncratic error between the software and hardware side given by $\sigma_\epsilon$ is close to 2. This implies that uncertainty or idiosyncracies in preferences over a software title is twice as large as the idiosyncracies over the non-software component of a hardware system. Although hardware is substantially a larger purchase decision in dollar value, it has little if any stand-alone value apart from its software; thus, any idiosyncratic utility a consumer derives from a hardware-software system is primarily contained in the consumer’s preference for different software titles. As a result, finding $\sigma_\epsilon > 1$ is not surprising.

Finally, $\sigma_\epsilon$ is also used to provide the ratio between mean hardware and software price sensitivities ($\alpha_{0,hw}^p = \sigma_\epsilon \alpha_{0,sw}^p$). The implied value of $\alpha_{0,hw}^p$ is reported at the bottom of the nonlinear parameter estimates. For robustness, I also estimated $\alpha_{0,hw}^p$ and $\sigma_{\alpha_{p,hw}}$ separately from software parameters, and could not reject the restriction imposed by the model. Although the standard errors on $\alpha_{p,hw}^p$ were larger when estimated separately (due to the lack of price variation in the hardware side), the proximity of this estimate to the value imposed by the restriction supports the link between the hardware and software demand systems.

**Linear Parameter Estimates**

The bottom portion of Table 3 reports linear hardware and software parameters $\theta_2 = \{\alpha^x, \alpha^w\}$. Software title fixed effects and seasonality effects are provided in the next table. The first immediate observation is the large difference in estimated fixed effect for the PS2 as compared to its rival platforms. Despite controlling explicitly for software, installed base, age, and seasonality effects, the PS2 hardware unit (net price) is estimated to be 3 times as valuable as its closest hardware competitor. Several factors included in the PS2’s fixed effect that are absent from its competitors include the ability to access and play the previous generation PS1’s existing library of over 1,000 games and the ability to play DVDs right out of the box.\(^{66}\) It may also reflect other factors such as the PS2’s unique hardware specifications, design aesthetic, or brand loyalty.

Unsurprisingly, the age of a console and software title is estimated to negatively affect lifetime utility from purchase. With hardware, it may partially reflect the fewer periods remaining to

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\(^{66}\)Although the GC could not play DVDs, the Xbox required users to purchase a separate accessory to enable playback.
enjoy the console before the next generation of video game systems are released (i.e., obsolescence), or merely some form of decay with respect to its perceived value, quality, or desirability; with software, the latter effects seem more likely to be the reason. On the other hand, the observed installed base of a product positively impacts utility for the Xbox and GC, but not the PS2. Recall that the coefficient on installed base reflects how lifetime utility is affected by the observed installed base; insofar that expectations of future installed base are correct and completely accounted for in each product’s fixed effect, the coefficient on installed base should be 0. As a consequence, the negative coefficient on the PS2’s installed base component may merely be a consequence that initial estimates for the PS2’s eventual installed base were much higher than what was observed, and over time perceptions (and hence, the utility) adjusted downwards despite an increase in the observed installed base. Similarly, a positive coefficient for the Xbox and GC may indicate an immediate increase in utility from more people coming onboard a system or using a software title, or may be a result from consumer’s revising their expectation over a product’s eventual installed base upwards over time.

Table 4 reports month fixed effects for hardware and software. As expected, the model predicts that seasonality effects dramatically influence when people purchase goods: holiday months exhibit highly positive and significant coefficients.

Table 5 presents an OLS regression of recovered software title fixed effects \( \{\alpha_k^w\}_{\forall k} \) on dummy variables indicating whether or not the title was exclusive (and if so, if it was published by a platform provider), the platform it was released on, and the month it was released. Results show exclusive titles that are published by platform providers (which are typically developed in-house) tend to exhibit higher quality than average. This is highly significant across both industry veterans – Sony and Nintendo – who have had each at least one generation of prior experience in software development, but is not significant for new entrant Microsoft. This may be evidence for integration as effort or quality enhancing for those firms with experience; on the other hand, the possibility that first-party titles are selected upon before being acquired or that first-party studios are simply higher quality game developers cannot be ruled out either. There is, however, a significantly negative coefficient across platforms for third-party exclusive titles. Consistent with industry stylized facts, most third-party titles that were exclusive did so not because they were compensated via some exclusive contract, but because the potential gains from multihoming would have been outweighed by the costs of porting to more consoles.

**Fit of Model**

In the appendix, I evaluate the fit of the estimated dynamic demand model, in particular focusing on assumptions 3.2 and 3.6 which govern the evolution of hardware mean-utilities \( \{\delta_{j,t}\}_{j \in J, \forall t} \) and product unobservables \( \xi, \eta \). I find the parameterization of the first-order Markov process \( F_i(\cdot) \) given by (19) provides a reasonable approximation of consumer expectations, monthly changes in product unobservable characteristics \( \nu^{hw}, \nu^{sw} \) are statistically uncorrelated, and the degree of multihoming by consumers predicted by the model is consistent with industry figures.
Alternative Specifications and Preliminary Counterfactuals

I now return to the alternate specifications listed in table 3. There are three dimensions along which the five specifications differ: dynamics, heterogeneity, and consumer multihoming. The static specification in (i) does not allow for any dynamic considerations, which include the persistence of unobservable characteristics, consumers leaving the market after purchase, and forward looking agents; unsurprisingly, it has the smallest likelihood and poorest fit. For the rest of the specifications, results in the table agree on the significance and relative magnitudes for most parameters in $\theta_1$ and $\theta_2$. At the same time, a standard likelihood ratio test rejects the hypothesis that there is no consumer heterogeneity across specifications, and that consumers can only buy one console. Where do the differences come into play? Tables 6 – 9 present different predictions that arise from alternative specifications of the model. Here, substantive differences emerge.

Table 6 reports own and cross-price semi-elasticities for platforms across three specifications. Each cell reports the percent change in market share of the platform located in the column due to a permanent 10% decrease in the price of the row-platform, where “Outside” indicates substitution to or from the outside good.\(^{67}\) The full model predicts larger differences within own market shares following a price drop compared to estimates from other specifications – i.e., a permanent 10% price drop in the GC results in a predicted increase of 25% in total consoles sold instead of 19% – yet a smaller effect on outside consumers. This occurs because in static model without multihoming, most of the increase in platform market shares from a price drop is predicted to come from consumers substituting away from non-purchase or from other consoles; the full model, however, predicts that many of consumers are not substituting, but rather purchasing an additional console.

Table 7 provides software own-price semi-elasticities for a representative “hit” title on each platform. These titles are the most popular titles on each console released in the first year of a console’s existence; thus, they were released when the selection by consumers onto platforms is most severe. It also happens that all three titles were exclusive to their respective system. The most striking difference in price elasticities occur across specifications which differ on consumer multihoming: without multihoming, a price drop results in a large increase in the number of consumers who purchase a title since it induces more people to purchase the console; with multihoming, this effect is significantly reduced (by over a half in the case of the Xbox and GC title) since many of those who might have purchased a console in order to access the title (i.e., high valuation consumers) would already have been predicted to own multiple consoles.

Table 8 presents changes in hardware installed bases if these three representative titles were not available – i.e., this provides an idea of the elasticity of demand with respect to a hit title.\(^{68}\) These elasticities can be extremely large: Microsoft’s *Halo*, in the full model, is predicted to have resulted in a 9% increase in the number of Xbox consoles sold (a difference of over 1.2M units). The dynamic specifications nearly double the predicted impact of a hit title on hardware demand

\(^{67}\)Since platforms are active for multiple periods, the price change is assumed to apply across the entire time period, and market shares are computed from installed base figures at the end of the sample period (October 2005).

\(^{68}\)I restrict attention only to losing the particular title, and not any sequels or titles in the same franchise.
compared to the static specification. This is not surprising: a static specification underestimates the quality of a product since it attributes non-purchase to low quality and not to people waiting for the price to fall. The addition of heterogeneity also impacts results: in its absence, a majority of consumers who substitute away from a console due to the loss of a hit title become non-purchasers of any console; however, with heterogeneity, these consumers instead are more likely predicted to substitute to another console. This is also intuitive.69 Capturing these dynamics is crucial in understanding how and why platforms compete for exclusive software.

I also examined what would have happened if the same hit titles for Xbox and GC had still been available on those platforms, but had also multihomed instead of being first-party exclusives. In other words, this is the benefit Xbox and GC expected to receive from exclusivity. The same differences across specifications was observed as before, with a static specification predicting less of an effect as the result of losing exclusivity, and specifications without consumer heterogeneity overestimating the impact of on non-purchasers. Nonetheless, I find that Xbox and GC would have been actually better off losing the title outright than having the title multihomed – e.g., had the hit title Halo for Xbox multihomed, Microsoft’s console would have seen its installed base fall by over 12% instead of 9% (since PS2 would have captured even more consumers from Xbox as a result). By acquiring the game studio that made Halo, Microsoft thus reaped significant gains in market share from exclusive access to the title.

Finally, in table 9, I explore a “naive” counterfactual environment in which all titles are forced to be available on all consoles, holding all other observed hardware and software prices and characteristics fixed. In such an environment, the differences across specifications are stark. All models without heterogeneity predict that when all titles are available on all videogame consoles, the vast majority of non-purchasers become hardware purchasers; with dynamics (but no heterogeneity), nearly every household is predicted to purchase a console. This is highly unrealistic – there are a significant number of consumers and households who, no matter what the availability of software may be, will not purchase a console due to income constraints or simply because they don’t value videogames. Introducing heterogeneity helps to correct for this out-of-sample prediction, and estimates that only approximately another 20-25M consumers would purchase consoles in the event of “forced compatibility.” Simulation runs also indicates that a model only allowing for consumer singlehoming underestimates both the benefit the PS2 receives from forced compatibility and the harm borne by the Xbox. Regardless of the specification, the counterfactual regime does seem to indicate that the PS2 does significantly better with the addition of new titles; the Xbox does significantly worse; and the GC does slightly better.

Nonetheless, this last counterfactual is hardly realistic; “forced compatibility” is not the appropriate regime to analyze when considering the absence of vertical integration or exclusive dealing.

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69 E.g., if all consumers were the same, then those who do not purchase a console when that console lost a title would be no different than those who elected not to purchase a console in the first place. However, with heterogeneity, those consumers who substitute away from purchasing a console because it lost the title are already more predisposed to gaming (since they would have purchased a console in the first place), and thus are much more likely to purchase another console instead.
The absence of these exclusive arrangements does not necessarily result in all titles multihoming across all consoles. In such a case, some titles still may voluntarily elect to be exclusive (or just support two consoles) since the costs of supporting more may be prohibitive. To address these concerns, I next focus on software provision and the decision of which platforms to support.

5 Hardware-Software Network Formation

For the consumer demand analysis, the set of software products on each platform has been conditioned on in the data. However when institutional features of an industry change -- as is the case when hardware platforms cannot vertically integrate into software provision or offer contracts contingent on exclusivity -- it is unlikely that the contracting relationships between parties will remain the same. In this section, I develop a model of software “demand” for hardware in order to determine how these relationships will change.

Recall each videogame software title needs to be specifically developed for a particular console in order to be used on that hardware system. If a title has agreed to an exclusive contract or is a first-party title, the decision of which platform to support has already been made; otherwise, a third-party title chooses the set of platforms that maximizes its expected profits. Each title weighs two competing forces when deciding which platforms to support: on one hand, developing and releasing a title for a platform provides access to that console’s base of users, which in turn may yield greater sales; on the other hand, such development requires the outlay of significant porting costs which may or may not be recouped.

The consumer demand system allows the econometrician to predict how many copies a title would have sold had it joined any set of platforms (including ones it was not observed to have developed for in the data), and how many more or fewer hardware consoles would have been sold as a result. However, a software title does not know exactly how many copies it will sell when it makes its decision: unlike a consumer who decides immediately which hardware platform or software title to purchase, a software developer must make a decision of which consoles to support 6 – 12 months in advance of release (in order to account for lead time in programming, development, and testing). The challenge is to use the demand system to construct estimates of each software title’s expected profits at the time a decision is made.

When evaluating the expected profitability of joining any set of platforms, a software title has expectations over not only its own quality and price upon release, but also the number of consumers that will be onboard each platform in the future. This introduces a significant complexity: in a platform market, a piece of software released for one platform induces more consumers to join that platform, which in turn may induce more titles to join, thereby driving more consumer adoption, and so on. Each software title thus must account for how consumers and other software titles act and react to all agents’ actions, including its own.

Nonetheless, by (i) leveraging the assumption used in the consumer demand analysis that software titles compete in independent markets and (ii) noting platform mean-utilities \( \{\delta_{j,t}\}_{j \in J, t} \) are
sufficient statistics for determining hardware demand, this particular issue can be resolved.\footnote{To save on notation, I assume that $\{\delta_{j,t}\}_{j \in J, t}$ includes hardware mean-utilities for all consumer types $i$ and inventory states $t$ (e.g., $\{\delta_{i,j,t}(i)\}_{i,j,t}$); and $\{\zeta_{j,k,t}\}_{j,k,t}$ includes software mean-utilities for all consumer types.} The first implies a software title is affected by the actions of other titles only if they affect the installed base of each console; the second implies that this can only occur through changes in $\{\delta_{j,t}\}_{j \in J, t}$. Together, they imply that beliefs over the evolution of $\{\delta_{j,t}\}_{j \in J, t}$ are sufficient for each title to internalize and condition on the future responses of consumers and other software titles. I assume each title perceives $\{\delta_{j,t}\}_{j \in J, t}$ to evolve according to the same first-order Markov process consumers believe; these beliefs are rational in that they are consistent with the realized empirical distribution of the underlying values. Furthermore, a title believes it impacts the level of $\delta_{j,t}$ if it joins platform $j$, but does not change its perception of the evolution of these mean-utilities.

The assumptions governing firm beliefs over the evolution of product mean-utilities are quite strong: though firms are assumed to share the same beliefs as consumers, these beliefs may be inconsistent in that the true evolution of $\{\delta_{j,t}\}_{j \in J, t}$ may not fall within the class of first-order Markov processes.\footnote{E.g., not only are entry and dynamic pricing decisions which enter into $\{\delta_{j,t}\}_{j \in J, t}$ not explicitly modelled, but expectations over future values of $\{\delta_{j,t}\}_{j \in J, t}$ may need to condition on more than what is contained in assumption 3.5.} At the same time, these assumptions are primarily made for tractability and allow for analysis without explicitly modelling the dynamic pricing and entry of new hardware or software products.\footnote{As long as all consumers and firms believe these values follow first-order processes fitted to their realized empirical distributions and optimize with respect to these beliefs, most agents undertake actions that are true best-responses. I conducted a robustness check by determining ex-post optimal actions for all software titles in a computed equilibrium, and found that they coincided with the equilibrium actions chosen by all titles of sufficiently high quality.} For the remainder of this paper, any equilibrium analysis will be conducted conditional on this restriction over agents’ beliefs.

The next subsection details the construction and computation of each software title’s expected profits. Since porting costs are unobserved, I focus on their estimation and recovery in subsection 5.2. In subsection 5.3, I present a dynamic game in which software titles are allowed to freely choose which platforms to develop for, and define and describe how to compute an associated equilibrium for the game. For the purposes of this analysis, I will assume that the decision of which platforms to join is made independently for each title, even if the title is released by a third-party publisher with multiple software products. Finally, in subsection 5.4, I test the fit of this model to the data by fixing the actions of all first-party titles, but allowing each third-party title to re-optimize and choose a new set of platforms. I find that the outcomes predicted by the model are very close to those realized in the data.

### 5.1 Software Expected Profits

Consider the decision faced by a particular third-party software title $k$ which will be released at time $r_k$. Assume $\tau$ months in advance, at time $r_k - \tau$, that title must choose a strategy $s_k \in S \equiv \{0, 1\}^3$ which indicates which set of the three platforms $k$ will develop for. For a given strategy $s_k$, title $k$’s expected discounted profits are given by (where, abusing notation slightly, $s_k$ also represents
the corresponding subset of platforms):

\[
E[\pi_k(s_k; \theta_C)|\Omega_{k,r_k-\tau}] = E\left[\sum_{t=r_k}^{T} 3^{t-r_k} \sum_{j \in s_k} Q_{j,k,t}((1 - \text{rmkup})p_{j,k,t} - mc_j)|\Omega_{k,r_k-\tau}\right] - C_k(s_k; \theta_C)
\]

(21)

where \(Q_{j,k,t}\) is the quantity of title \(k\) sold on platform \(j\) at time \(t\), \(\text{rmkup}\) denotes the markup captured by retailers, \(mc_j\) is the marginal cost of production on console \(j\) (which includes royalties paid to the platform provider), and \(C_k(s_k; \theta_C)\) are the costs of producing title \(k\) for all the platforms within \(s_k\) which depends on some vector of parameters \(\theta_C\). In addition to development and programming costs, \(C_k(\cdot)\) contains all other fixed costs related to the production of the game including distribution and marketing. These costs are known to the software title at time \(r_k - \tau\) but not to the econometrician. Finally, expectations are conditional on \(\Omega_{k,r_k-\tau}\), software title \(k\)'s information set at time \(r_k - \tau\), which includes any factors affecting market characteristics and consumer demand.

From the demand system, \(Q_{j,k,t}\) can be computed for any title at time \(t\) conditional on knowing its quality and price, and the installed base of consumers onboard platform \(j\) who have not yet purchased \(k\). To compute the expected value of \(Q_{j,k,t}\) in future periods, I assume every software title knows its initial price and quality at the time of release. I also make the following assumptions:

**Assumption 5.1.** For each inventory state \(i \in I\), software titles perceive \(\{\delta_{i,j,t}(\cdot)\}_{\forall j \in J_{i,t}}\) can be summarized by first-order Markov processes \(F = \{F_i(\cdot;i)\}_{\forall i,t}\) given by (14).

**Assumption 5.2.** Each software title perceives its own \(\{\zeta_{i,j,k,t}\}_{\forall i,j \in J_{i,t}}\) can be summarized by first-order Markov processes \(\{G_{j}\}_{\forall j} \equiv \{G_{i,j}\}_{\forall i,j}\) given by (10).

These are the parallels to assumptions 3.4 and 3.5 used for consumer demand analysis, and imply that software titles share the same beliefs as consumers over the evolution of both \(\{\delta_{j,t}\}_{j \in J_{i,t}, t > r_k - \tau}\) and \(\{\zeta_{j,k,t}\}_{j \in J_{i,t}, t > r_k - \tau}\). With these assumptions, as long as title \(k\) knows the transition probabilities \(F(\cdot)\) and \(\{G_{j}(\cdot)\}_{\forall j}\), then the month, the installed base of consumers on each platform at time \(r_k - \tau\), hardware mean-utilities \(\{\delta_{j,r_k-\tau}\}_{j \in J_{i}}\), and the title’s own starting qualities \(\{\zeta_{j,k,r_k}\}_{j \in J_{i}}\) and price path are all that are required to compute the number of copies \(\{Q_{j,k,t}\}_{j \in J_{i}, t \geq r_k}\) each title expects to sell on each platform. In the appendix, I detail the associated computational routine required to recover estimates of these quantities and hence each title’s expected profits (given costs).

For estimation, I will assume the retailer markup is fixed at 35% and marginal costs are constant across platforms at $10 (reflecting royalty rates of approximately $7 and production costs of $3 per game disc). These figures are consistent with information provided by industry and public sources.\(^{73}\)

### 5.2 Recovery of Development and Porting Costs

In order to compute a title’s expected profits from choosing any particular action \(s_k\), one final issue remains: development and porting costs \(C_k(\cdot; \theta_C)\) are unobserved. To estimate and recover these

\(^{73}\)See e.g. Takahasi (2002).
unobserved costs, I use a methods of moments estimator based on inequality constraints developed in Pakes, Porter, Ho, and Ishii (2006).\footnote{See Ho (2007) and Ishii (2005) for other applications. Unlike using a multinomial logit model to determine costs based on observed actions for each software title, this procedure can handle the correlation of certain kinds of measurement and expectational errors without needing to fully specify their joint distribution. Furthermore, it imposes fewer assumptions than would otherwise be required.}

Consider again the decision of a third-party title \( k \) who decided \( \tau \) months in advance of release which platforms to develop for. The key assumption used to generate the moments for estimation is that for each title brought to market by a third-party publisher, the expected profits from developing for the set of platforms it chose in the data were higher than developing for any other set of platforms, holding fixed the actions for all titles released up to that point in time:

**Assumption 5.3.** For each third-party software title \( k \), the observed choice of platforms \( s^o_k \) maximized its expected profits:

\[
E[\pi_k(s^o_k; \theta_C)|\Omega^o_{k,r_k-\tau}] \geq E[\pi_k(s'_{k}; \theta_C)|\Omega^o_{k,r_k-\tau}] \forall s'_{k} \in S
\]

where \( \Omega^o_{k,r_k-\tau} \) denotes the observed state of each title’s information set at time \( r_k - \tau \).\footnote{I will assume that the econometrician’s estimate and a title’s estimate of expected profits are the same. As long as the error between the two is mean zero across titles and strategy choices and independent of instruments chosen, the following analysis does not change. This error is the “\( \nu \) error” in Pakes, Porter, Ho, and Ishii (2006).}

I use a parsimonious specification for \( C_k(s_k; \theta_C) \):

\[
C_k(s_k; \theta_C) = c_0(s_k) + \sum_{j \in s_k} c_j \alpha_{0,j,k}^w + \nu^c_k
\]

where \( \alpha_{0,j,k}^w \) represents the software fixed effect for title \( k \) on platform \( j \) perceived by the mean consumer (estimated from the demand side), and \( \theta_C \equiv \{ \{ c_0(s) \}_{s \in S \setminus \{0\}^3}, \{ c_j \}_{j \in J} \} \). \( \nu^c_k \) represents title-specific costs that affect all strategy choices equally. The difference in costs between two different titles are thus assumed to be contained within differences in the estimated software fixed effect and some unobservable title-specific component.

Given assumption 5.3 and the specification of porting costs given by (22), the expected difference in profits between the observed strategy chosen and any alternative should be positive for all titles:

\[
E_k \left[ E[\pi_k(s^o_k; \theta_C)|\Omega^o_{k,r_k-\tau}] - E[\pi_k(s'_{k}; \theta_C)|\Omega^o_{k,r_k-\tau}] \right] \geq 0 \forall s' \in \{ S \setminus \{0,0,0\} \}
\]

Since I do not observe software products which are not released on any platform, I restrict attention to strategies that involve joining at least one platform.

Let \( K_s \) denote the set of titles that choose strategy \( s \). For each \( s \neq \{0\}^3 \) and \( s' \notin \{ s \cup \{0\}^3 \} \), converting expectations into sample means yields the following inequality moments:

\[
\frac{\sqrt{\#K_s}}{\#K_s} \sum_{k \in K_s} (E^o[\pi_k(s; \theta_C) - E^o[\pi_k(s'; \theta_C)]) \otimes g(\omega_{k,t-\tau}) \geq 0 \tag{23}
\]
for any $\omega_{k,r_k-\tau} \in \Omega_{k,r_{k-\tau}}$, where $\otimes$ represents the Kronecker product and $g(\cdot)$ is any positive valued function. I weight by the square root of the number titles that choose each particular strategy $s$ in order to account for the fact that there should be less expectational noise in expected profits for strategies chosen by many titles.

Equation (23) defines 42 inequalities (7 non-zero strategies, each with 6 alternative strategy comparisons) to be used in estimation for each choice of instrument. If there are multiple values of $\theta_C$ which satisfy the inequalities, all values are admissible and a set estimate is provided; otherwise, the value $\hat{\theta}_C$ which minimizes the absolute value of deviations in the inequalities is obtained.\textsuperscript{76}

Since only strategies that involve joining at least one platform are compared, all components in $\theta_C$ are not identified: only the relative differences between $c_0(s)$ and $c_0(s')$ can be determined. Nonetheless, for the purposes of the subsequent analysis, relative differences are all that are required in order to determine the optimal choices for software titles. In estimation, $c_0(\{1,0,0\})$ (i.e., the constant cost for developing only for the PS2) is fixed to be 0.

Estimates

Table 10 presents porting cost estimates for $\tau = 6$ (i.e., software titles make their decision 6 months prior to release). Since the costs for developing solely for the PS2 are fixed to be 0, these estimates reflect the relative costs of porting to a particular set of consoles.

The specification reported allows costs to vary across different software genres. Depending on the genre, certain consoles may be more difficult to develop for than others, or higher quality games may be more or less difficult to produce. The results confirm such variation.

For the average title, results indicate developing for two consoles is generally more expensive than developing for one, but still cheaper than developing for all three; and developing for the Xbox and GC is cheaper than doing so for either the PS2 and GC or the Xbox and GC. Aggregating across genres, porting an average quality title to an additional console can range from approximately $300K to $2M depending on the console chosen and set of instruments used. These are in line with figures provided by industry sources. Furthermore, for most titles, Xbox and GC are to be significantly cheaper to develop for than the PS2 – this is also consistent with institutional details.\textsuperscript{77}

Repeating the exercise for different values of $\tau$ (including 9 and 12 months) did not significantly affect estimates of porting costs, nor affected the counterfactual analysis that follows. The choice of instruments, however, did affect point estimates and associated confidence intervals. To account for this variation, I used several different sets of instruments and associated porting cost estimates.

\textsuperscript{76}When constructing the inequality estimators, I also omit “high-quality” exclusive titles brought to market by third-party publishers, which I assume to be those with estimated fixed-effects $a_{0,k}^{w} > -3$. The reason for this restriction is that these exclusive titles, although not first-party, may have been subject to unobserved exclusive deals involving royalty rate reductions, lump sum payments, development assistance, or joint marketing promotions. The underlying assumption is that all other titles – those that multihomed were of low enough quality – did not receive any exclusive contracts or preferential treatment from console providers. Because of the specification of costs in (22), any title specific errors are differenced out and no selection problem is introduced.

\textsuperscript{77}E.g., the PS2 with a new CPU architecture had a reputation of being difficult to develop for, whereas the Xbox was essentially an Windows-Intel PC using APIs many developers were already familiar with.
when computing confidence intervals for all of the following counterfactual exercises. As will be evident, the predictions and results were still fairly robust to the choice of instruments used.

5.3 Dynamic Network Formation Game

I now specify a dynamic network formation game in which each software title prior to release selects which platforms to develop for having formed expectations over the future profitability of each potential strategy. The setup allows for contracting partners and consumer demand to change over time with past actions influencing future decisions, a crucial feature to capture in this networked industry. Of course, other industry features may change as well, including the investment incentives which will affect entry and exit of new hardware and software products. For the purposes of expositing the model, I will focus here only on changes in contracting partners, assuming that the set of available hardware and software products is given, and porting costs, royalty rates, retailer markups, and release dates do not change. In the next section when I estimate a counterfactual regime, I discuss potential ways of relaxing some of these restrictions.

Setup and Timing

In each period \( t \), there is a set of software products \( K_t^R \) that will be released on at least one console. At time \( t - \tau \), every title \( k \in K_t^R \) simultaneously commits to a set of consoles \( s_k \) that it will be released on \( \tau \) periods in the future. This decision is private, and is not observable or known to any other industry participant until the title is released at time \( t \). Each title observes the number of each type of consumer both off and onboard each platform, knows its own initial quality \( \zeta_{k,t} \) and release price \( p_{k,t} \), but does not have any information about future software releases or availability \( \{K_t^R\}_{t' \geq t - \tau} \). Finally, platforms have no strategic actions, and each offers a fixed contract to all titles specifying a common royalty rate.\(^{79}\)

At each period \( t \), the timing of actions is as follows:

1. All titles \( k \in K_t^R \) are released and added to the stock of existing software products for each platform according to \( \{s_k\}_{\forall k \in K_t^R} \);
2. Characteristics for all platforms and released software titles are determined (which are subsumed by \( \{\delta_{j,t}\}_{j \in J_t} \) and \( \{\zeta_{j,k,t}\}_{\forall j,k \in \{\cap_{t' \leq t} K_t^R\}} \));
3. Consumers make hardware and software purchase decisions;

\(^{78}\)Note there is no incomplete information in this game: even though each software title’s action and quality is observable only after a \( \tau \) period delay, neither affects any other agents’ payoffs until they are perfectly revealed (i.e., when the title is ultimately released).

\(^{79}\)Platforms are not assumed to be strategic agents other than setting the prices for their own consoles, which is internalized in the evolution \( \{\delta_{j,t}\}_{j \in J_t, \forall t} \). Without exclusive deals or vertical integration, I rule out any preferential treatment by platform providers towards individual software titles since these deals are primarily made in exchange for exclusivity. Additionally, platforms typically pre-announce and commit to royalty rates charged to third-party software developers in advance of a system’s release,\(^{80}\) and I assume that these royalty rates not only do not change, but remain the same in counterfactual environments.
4. All titles \( k \in K_{i+\tau}^R \) commit to join platforms \( j \in S_k \).

Equilibrium and Computation

Given assumptions 3.4, 3.5, 5.1 and 5.2 on consumer and firm beliefs over the evolution of hardware and software mean-utilities, a first-order Markov equilibrium of this game will contain a set of strategies \( \hat{s}_k \) and first-order Markov transition processes \( \hat{F} \) and \( \{\hat{G}_j\}_{vj} \) such that:

1. For every title \( k \), \( \hat{s}_k \) maximizes its expected profits at time \( r_k - \tau \):

\[
E[\pi_k(\hat{s}_k; \theta_C)|\Omega_{k,r_k-\tau}] \geq E[\pi_k(s'_k; \theta_C)|\Omega_{k,r_k-\tau}] \forall s'_k \in S \setminus \{0,0,0\}
\]

with each title’s expected profits given by (21), and beliefs over the evolution of \( \{\delta_{j,t}\}_{j \in J_t, vt} \) and \( \{\zeta_{j,k,t}\}_{vj,j,t>r_k} \) given by \( \hat{F} \) and \( \{\hat{G}_j\}_{vj} \);

2. Consumers purchase hardware and software according to the dynamic model provided in section 3, with software availability given by \( \{\hat{s}_k\}_{vk} \) and beliefs over the evolution of \( \{\delta_{j,t}\}_{j \in J_t, vt} \) and \( \{\zeta_{j,k,t}\}_{vj,j,t>r_k} \) given by \( \hat{F} \) and \( \{\hat{G}_j\}_{vj} \);

3. Transition processes \( \hat{F} \) and \( \{\hat{G}_j\}_{vj} \) are consistent with realized values of \( \{\delta_{j,t}\}_{j \in J_t, vt} \) and \( \{\zeta_{j,k,t}\}_{j \in J_t, k \in K_{j,t}, vt} \) implied by \( \{\hat{s}_k\}_{vk} \) and consumer behavior.\(^{81}\)

In this equilibrium, each software title conditions only on its own mean qualities, prices, and other “payoff-relevant” state variables when determining its optimal strategy;\(^{82}\) furthermore, a consumer’s decision to purchase a particular platform or software title – as on the demand side – is only a function of her own characteristics and the product’s mean-quality (\( \delta_{j,t} \) or \( \zeta_{j,k,t} \)). A first-order Markov equilibrium is thus a Markov-Perfect Nash Equilibrium in the sense of Maskin and Tirole (1988, 2001) with the additional restriction that agents’ beliefs over the transition probabilities governing \( \{\delta_{j,t}\}_{j \in J_t, vt} \) and \( \{\zeta_{j,k,t}\}_{j \in J_t, k \in K_{j,t}, vt} \) are contained within the class of first-order Markov processes.

It is worth stressing that this equilibrium is indeed subgame perfect (again, restricting agents’ beliefs to take the form of first-order Markov processes): as long as every agent chooses its optimal action as a function only of its own payoff-relevant state variables as in the specified equilibrium, any software title’s action \( \hat{s}_k \) or consumer’s purchasing decision remains optimal and is a best-response even when considering more general deviations (e.g., non-Markovian strategies such as conditioning choices explicitly on the previous actions of other agents).\(^{83}\)

Finally, it may be the case that there are multiple equilibria: different beliefs over the evolution of each hardware platform’s quality may sustain different actions which in turn rationalize those

\(^{81}\)These transition probabilities will likely be different from those originally estimated in the demand system since they account for any changes in contracting partners from those observed in the data.

\(^{82}\)As discussed earlier, since each software title is assumed to compete in independent markets its profits in each period depend only on: its own mean-utilities \( \{\zeta_{j,k,r_k}\}_{j \in J_t} \), prices and costs; the installed base and mean-utility \( \{\delta_{j,t}\}_{j \in J_t} \) of each platform; and the time of year.

\(^{83}\)This follows directly from the formulation of each consumer’s dynamic programming and title’s maximization problem.
beliefs. In computation, the space of beliefs are restricted to the parameterizations of $F$ and 
$\{G_j\}_{\forall j}$ used on the demand side, given by (19) and (20); additionally, there are bounds on the set
of sustainable beliefs in equilibrium since there are minimum and maximum attainable values of
$\{\delta_{j,t}\}_{j \in J_t, \forall t}$ for each platform, which correspond to hardware mean-utilities without any or with
all software titles onboard. Yet there still may exist multiple equilibria that satisfy the conditions
above. To partially account for this possibility, I run the following computation algorithm with
different starting beliefs:

- I first fix the transition processes governing the evolution of $\{\delta_{j,t}\}_{j \in J_t, \forall t}$ and $\{\zeta_{j,k,t}\}_{j \in J_t, k \in K_{j,t}, \forall t}$ to starting beliefs $F^0$ and $\{G^0_j\}_{\forall j}$. For robustness, I use 5 different sets of starting beliefs $F^0$ which govern the evolution of hardware qualities $\delta$: one which assumes no software title joins any console, one which all titles join every console, and three different sets in which all
titles join only one console.

- In each iteration $n$, I proceed forward from $t = 0$ and at every period: update $\{\delta_{j,t}^n\}_{j \in J_t, \forall t}$ for each console based on the set of new titles released and their chosen strategies; evaluate consumer demand over the set of hardware and software products; and compute the optimal
strategy $s_k^t$ for each title $k \in K_{t+\tau}$ to be released $\tau$ months in the future.

- After the optimal actions for all titles in every period are computed, I use the implied paths of $\{\delta_{j,t}^n\}_{j \in J_t, \forall t}$ and $\{\zeta_{j,k,t}^n\}_{j \in J_t, k \in K_{j,t}, \forall t}$ to update the transition processes according to (19) and (20), obtain new estimates for $F^{n+1}$ and $\{G_j^{n+1}\}_{\forall j}$, and repeat the simulation until
no software title changes its chosen action from the previous iteration and the estimated transition processes converge.

5.4 Fit of Dynamic Network Formation Model

To evaluate how well this model predicts the strategic actions of firms, I compute a new equilibrium
holding fixed the actions of all first-party software titles, but allowing every third-party title to re-
optimize and choose a new set of platforms to support. Table 11 presents a comparison between
the observed data and this new equilibrium. Confidence intervals are constructed by repeating the
simulation using draws from the distribution of estimated porting costs in the previous section for
multiple sets of instruments.

The model predicts both installed base figures and market shares for each console to be very
close to those observed in the data. Although the PS2 is predicted to have fewer titles join and
the Xbox and GC more, restricting attention to only “hit” titles – defined as titles selling over
100K or 1M copies on a given console – indicates a much closer fit. This indicates that although
the estimation error or specification error in porting costs may likely affect the actions chosen by
small titles, it becomes less of an issue for the larger, more popular games. Since the actions of
these larger titles are the only ones that dramatically affect platform market shares, as long as their
actions are accurately predicted, the estimated aggregate industry figures such as market shares
and installed base figures will be similar to those observed.
Finally, the total number of titles sold on each platform is also provided, and again the model predicts figures close to those given by the data. These totals are important since most of each platform’s profits are derived from royalty payments on software, not from hardware sales; they translate directly into the profitability and success of each platform.

In all simulated runs, using different starting beliefs did not change the computed equilibrium. This is due to the fact that for each title, the decision of which consoles to support is typically robust to small fluctuations in beliefs over the evolution of \( \{\delta_{j,t}\}_j \in J, \forall t \). Only “high quality” titles can shift the value of \( \{\delta_{j,t}\}_j \in J, \forall t \) for any given console in any significant way, and the strategy of these titles is not significantly influenced by the strategies of other software titles. Indeed, recall most of a title’s sales occur in the first 3 months of release, and as long as there are sufficient numbers of consumers on board each platform at a given moment, most hit titles will multihome across all platforms. On the other hand, for most mid and low-quality titles, the first-order impact of porting costs on profits typically dwarfs the impact of any small shifts in expected installed bases across consoles caused by changes in beliefs, and these titles will typically only choose one or two platforms to develop for.

6 Policy Experiment: Banning Vertical Integration and Exclusive Contracting

Using the framework developed in and estimates obtained from previous sections, I proceed in this section to examine a counterfactual environment in which console providers are prevented from integrating into software development, and are unable to offer exclusive contracts to third-party titles. However, third-party titles still may voluntarily choose to be exclusive if they find the costs of porting too high.

As mentioned before, a change in the contracting space eliminating contractual exclusivity may also have affected the entry or exit of software products. For example, without integration investment in previous first-party software may not have occurred. To account for this possibility in the counterfactual exercises, I consider two cases: (i) first-party titles are assumed to still enter the market as third-party titles; (ii) all first party titles no longer are produced and thus do not enter. These two alternatives can be used as potential proxies for what actually may occur absent exclusivity and integration.

Some caveats remain. Although software providers form expectations over the future quality and prices of products when deciding which platforms to join, I assume prices are the same as actual price paths in the data when computing the outcomes of the counterfactual regimes. Without a full model governing the dynamic price setting behavior of firms, these counterfactual price paths become difficult to determine.\(^{84}\) Furthermore, I assume that all platforms offer the same non-

\(^{84}\)Nair (2007) provides one approach to modelling a firm’s dynamic pricing decision. Nesting this type of optimization problem within this paper’s dynamic hardware and software demand system while still accounting for the endogenous selection of heterogeneous consumers across platforms is too computationally burdensome for implementation. Nonetheless, I also compute the counterfactual simulations assuming software prices follow a first-order
exclusive contracts to each software title and do not change their royalty rates.

6.1 Industry Structure

The results from the counterfactual simulations are presented in table 12. Generally, more titles are predicted to multihome and port than observed in the data, which is expected. In the first specification when former first-party titles still enter the market as third-party titles, the change in market shares is stark: Sony’s PS2 is predicted to command over 75% of the market by October 2005 as opposed to half, and sells nearly double the number of consoles and nearly five times as many total copies of software as before. On the other hand, Microsoft’s Xbox does significantly worse, selling 5M fewer consoles and nearly half as many copies of games. The relative success of Nintendo’s GC is less clear; it loses market share and sells approximately the same number of hardware devices, but does manage to sell more software titles. The reason that the response of the Xbox and GC are different is because the Xbox, which sells at the same price point as the PS2, is more of a direct competitor and substitute for the PS2. Without exclusive titles distinguishing the PS2 from the Xbox, most consumers will select the PS2 due to its higher predicted fixed effect and 14 month head start in accumulating a larger installed base and software library. However, consumers may still flock to the GC because of its cheaper price – it appeals to price sensitive consumers as a first console, and also to other owners of the PS2 or Xbox for its titles that may still be voluntarily exclusive.

The second specification conducts the same exercise, except now removes all former first-party titles from consideration. The industry as a whole does worse since most of the first-party titles were blockbuster hits; nonetheless, because the PS2 still captures a majority of the market and enough software products, it still does better than it did originally. Here, both the Xbox and GC are predicted to do significantly worse, with each selling far fewer consoles and software titles than they did in the presence of exclusivity. Recall table 2 showed nine of the top ten titles sold on GC were first-party titles, and that table 5 indicated the first party titles on GC were shown to be much higher quality on average; if the inability to vertically integrate leads to the absence of these former first-party titles, then it is unsurprising that the total number of titles sold on the GC is predicted to drop precipitously in their absence.

6.2 Consumer Welfare

Without exclusive arrangements, consumers onboard each platform would have access to a larger selection of high quality titles. To analyze consumer welfare gains, I calculate the compensating variation for consumers who are predicted to purchase a console in each counterfactual environment.

In the first regime, when first-party titles still enter the market, I find that total consumer welfare increases by approximately $7B holding fixed hardware and software prices and entry. Half of this increase is realized by those consumers who would have purchased a videogame console in the

Markov process estimated in the appendix, and find that results do not change significantly.
previous regime; these consumers receive on average approximately $72 more in surplus. The other half of consumer welfare increase is realized by new consumers who previously did not purchase a console, but now would; 26M new households on average receive approximately $140 in surplus. However in the second regime when first-party titles are no longer produced, consumer welfare is predicted to increase only by $.7B with existing purchasers gaining on average only $10, and new purchasers (of which there are far fewer) approximately $80. The loss of first-party titles, thus, drastically reduces the potential gain from increased software compatibility.

Both of these calculations again ignore the possibility that Microsoft or Nintendo may have exited the market, or Sony, with its increased market power, may have increased prices. For example, in the first regime if Microsoft and Nintendo did not enter at all due to the inability to integrate or obtain exclusive titles, total consumer welfare would be predicted to have fallen by $1B despite having all titles onboard a single console. Similarly, I find Sony – holding the prices fixed for the Xbox and GC – would have found it profitable to raise the price of its PS2 unit by over $200, again eliminating most consumer welfare gains in either counterfactual regime.

6.3 Discussion and Policy Implications

Counterfactual experiments suggest that vertical integration and exclusive contracts were generally pro-competitive at the platform level, benefiting Microsoft and Nintendo and helping them establish a foothold into the sixth-generation videogame market. Since the PS2 had already captured an installed base of 5M users before its two competitors entered a year later, without exclusive arrangements the Xbox and GC would likely only have been able to induce a developer to release a title for their consoles after a version had already been developed for the PS2. Hence, neither entrant would have been able to obtain any software advantage over the incumbent.

Furthermore, with Sony predicted to command over three-quarters of the market in the absence of exclusive contracts or integration, it is likely that it could have sustained higher prices – although it may not necessarily have charged a higher introductory price, Sony would nevertheless not have been as hard pressed to anticipate or match Microsoft’s and Nintendo’s price cuts. Combine this with the possibility that Microsoft and Nintendo may not have found it profitable to enter or remain in the videogame industry and subsequently not produced their seventh-generation consoles, any immediate consumer gains from increased access to software may very likely have been offset by these dynamic consequences of monopolization. Although these welfare calculations are sensitive to the assumptions used concerning entry, exit, and price setting behavior, the implications governing industry structure, market concentration, and platform competition appear to be robust.

It is worth stressing that although exclusive arrangements may have encouraged platform competition, this does not necessarily imply that they encouraged software competition. In the videogame industry, without modelling the entry and exit of new titles, the effect on software is ambiguous – having only a single monopoly platform to support might have reduced porting costs required for a third-party developer since only one console would have to be developed for, but a more competitive environment with multiple integrated platform providers might have increased investment in
first-party software or led to fiercer competition among platforms for titles through reduced royalty rates or increased development and marketing assistance.

In other industries, the impact of vertical integration and exclusivity on software competition is more clear. For example, consider Microsoft’s integration of its platform (Windows OS) into the browser and media application space. The courts in both *U.S. v. Microsoft* and *European Union v. Microsoft* ruled that these actions resulted in foreclosure of competing software vendors (e.g., Netscape and Real Networks). Although both the videogame industry and the PC industry are similar hardware-software environments, the fact that PC applications are very close substitutes for one another (consumers typically only use one word processor, browser, media player, or spreadsheet program), whereas videogames are not (buying one action game does not preclude the purchase of another), indicates that “upstream” software foreclosure may be more of a concern when such substitutability is an issue. The results of this analysis, if extended to the PC industry, would indicate that vertical integration and exclusivity into software development may aid other platform providers (such as MacOS and Linux) in competing against Windows, but potentially foreclose third-party software and application developers from entry.

A more suitable comparison to the videogame industry is in television distribution, specifically with regards to competition between satellite and cable television providers for exclusive content. In the U.S., DirecTV’s exclusive contracts with certain content providers – notably with the National Football League for a package of its out-of-market games – substantially contributed to its success and ability to induce consumers to substitute away from cable. The impact of this competition was substantial: Goolsbee and Petrin (2004) estimate that entry by satellite providers reduced cable prices by about 15% and encouraged improvements in cable quality, yielding aggregate welfare gains of approximately $5B. In this regard, the U.S. Senate’s recent actions preventing an exclusive deal between Major League Baseball and DirecTV – as detailed in footnote 8, an intervention motivated mainly by a static efficiency desire to expand consumer access – may have negatively affected competition in the industry. Without these exclusive carriage deals, cable providers as well as other distribution channels in similar media markets may face less competitive pressure.

At the same time, in other industries platform competition may not be desirable and monopolization actually preferable. Often this is true when a platform provider cannot raise prices or otherwise exercise market power upon establishing a dominant position. For example, consider the two main competing standards for next generation DVDs – Blu-ray and HD-DVD, sponsored primarily by Sony and Toshiba, respectively. Here, even if one standard wins and “monopolizes” the market, that platform sponsor cannot increase prices since it has already committed to licenses and royalty rates with hardware manufacturers and movie studios. Furthermore, having a single clear standard emerge as the dominant platform will effectively remove uncertainty from the marketplace and likely spur consumer adoption, thereby increasing total welfare.\(^{85}\) As a consequence, integration and exclusive contracting between the standard sponsors and motion picture studios (e.g., Sony’s ownership of Columbia Pictures or Toshiba’s exclusive deal with Paramount) – although

\(^{85}\) See, e.g., Farrell and Saloner (1985) and Katz and Shapiro (1986) for discussion on the benefits of standardization.
“pro-competitive” in encouraging the existence of multiple formats according to the analysis of this paper – may actually be contributing to an undesirable and lengthy standards battle.

7 Concluding Remarks

This paper has shown that integration and exclusive contracting between console manufacturers and software developers in the videogame industry likely encouraged and enabled platform competition. Evidence suggests that in this and other platform markets where upstream foreclosure is not a concern and where forced exclusivity contracts are not permitted, intervention or regulation may not be necessary. Furthermore, when evaluating the possibility of foreclosure or entry deterrence in dynamic networked environments, traditional static analysis may fail to uncover significant pro-competitive effects of exclusive vertical arrangements.

Although counterfactual simulations indicate that the industry is far more competitive when exclusive arrangements are allowed, consumers may still have benefited from access to a wider selection of software titles onboard each platform if these arrangements were prohibited. Whether or not consumers would have been better or worse off as a whole depends crucially on whether the incumbent would have raised prices in response to its increased market power, and whether competing platform providers or software titles would have exited. Any of these effects would have been sufficient to eliminate potential consumer welfare gains.

This paper also focused on developing a framework to empirically measure the impact of these exclusive arrangements. I presented a consumer demand system that accounts for the dynamic selection of forward-looking, heterogeneous consumers across and onto platforms; the demand system also can recover the contribution of an individual title to any given platform. Additionally, I detailed and estimated a computationally tractable dynamic network formation game which allows agents to anticipate the future actions of other players by conditioning on a small dimensional set of state variables. By thus explicitly modelling the platform adoption decisions of individual consumers and firms, the framework here can be applied to structurally analyze other related industries that exhibit similar indirect network effects.

To make analysis tractable, a few key assumptions were made and carried throughout the paper. First, I assumed the independence of software titles; in other types of platform industries, there may be stronger substitution effects across affiliated products. Secondly, to model the dynamic decisions for both consumers and firms, I restricted agents’ beliefs over the evolution of product mean-utilities to the class of first-order Markov processes. Finally, I abstracted away from product entry and exit decisions as well as the dynamic pricing problem faced by both hardware and software providers. Some robustness checks and alternative formulations were explored. Though it is unlikely that relaxing these assumptions would change the counterfactual implications of this paper, it may be necessary for other empirical applications and is the subject of future work.
A Demand System: Further Details

This section of the appendix provides further details on the specification, estimation, computation, and robustness checks for the consumer demand analysis.

A.1 Multiple Hardware Purchases

To reduce the number of options a consumer has at any period in time, I will assume that a consumer can purchase at most one console per period, will never purchase a console she has already bought, and can return in any future period to purchase another console. The consumer’s value function for being able to purchase a new console is thus given by:

$$V_i(\epsilon_{i,t}, \Omega_{i,t}) = \max \{ \max_{j \notin \epsilon_i} u_{i,j,t}(\epsilon_{i,t}, \omega_{i,t}(\Omega_{i,t})) + \beta E[V_i(\epsilon_{i,t} \cup \{j\}, \epsilon_{i,t+1}, \Omega_{i,t+1} | \Omega_{i,t})], u_{i,0,t}(\epsilon_{i,t}) + \beta E[V_i(\epsilon_{i,t}, \epsilon_{i,t+1}, \Omega_{i,t+1}) | \Omega_{i,t}] \} \tag{24}$$

That is, a consumer will either choose to purchase or not purchase a new console, and if she does decide to buy, she will purchase the one that delivers the highest expected lifetime utility. In either case, she accounts for the continuation or option value of remaining on the market and updates her inventory state depending on her chosen action.

Using assumptions 3.1 and 3.5, (24) can be integrated over $\epsilon$ and rewritten as:

$$EV_i(\{\delta_{i,j,t}(\epsilon_{i,t})\} j \in J, \epsilon_{i,t}, m(t)) = \int_{\epsilon_{i,t}} V_i(\epsilon_{i,t}, \epsilon_{i,t}, \Omega_{i,t}) =$$

$$\ln \left( \sum_{j \notin \epsilon_{i,t}} \left( \exp(\delta_{i,j,t}(\epsilon_{i,t}) + EV_i(\{\delta_{i,j,t}(\epsilon_{i,t}) \cup \{j\}\} j \in J, \epsilon_{i,t} \cup \{j\}, m(t+1))) + \exp(\beta E[EV_i(\{\delta_{i,j,t+1}(\epsilon_{i,t})\} j \in J, m(t+1)|\{\delta_{i,j,t}(\epsilon_{i,t})\} j \in J, m(t)) \right) \right) \tag{25}$$

The only remaining issue is that the equations which govern the predicted share of consumers which purchase a console must now integrate over the distribution of not only consumer types, but consumer inventory states as well. Accounting for this as well as how the distribution of consumer inventories changes over time is detailed when computational details are discussed in A.3.86

A.2 Other Industry-Specific Issues

There are a few remaining institutional specific details that affect the estimation of the model.

The first involves an issue of staggered platform release dates: Sony’s PS2 console was released in October 2000, whereas the Nintendo GC and Microsoft Xbox were not released until November 2001. Consequently, for a portion of the data, only one console was available. Nonetheless, consumers knew that the GC and Xbox would be released during the 2001 holiday season even a year in advance of the actual release.87 As a result, I model the consumer’s relevant problem from October 2000 to October 2001 as a finite horizon optimal stopping problem with only one hardware console available, and assume that consumers know the starting values for the new products when they are introduced in November 2001.

The second is that the PS2 is backwards compatible with titles released for Sony’s previous generation console, the original Playstation (PS1). Any utility derived from titles released for the PS1 prior to October 2000 as well as expectations over future software availability would be subsumed in the PS2’s fixed-effect; however, any unexpected utility from PS1 titles released afterwards would not be accounted for. From the release of the PS2 in October 2000, there were 387 titles released for the PS1, 332 of which were not also released for the PS2. None of these were large successes. Since it is impossible to differentiate whether or not purchasers of these software titles owned a PS1, 86A final concern governs the software purchase decision of consumers who own multiple consoles – a consumer who has already purchased a title on one console she owns should not typically also purchase the same title on a second console. Unfortunately, correcting for this would involve tracking the inventory of software purchases for each consumer, which is not computationally feasible.

87Microsoft officially announced the Xbox on March 10, 2000 and Nintendo announced the GC on August 25, 2000, although their existence was rumored for months prior. As often is the case, console manufacturers announce the upcoming release of a new console far in advance to drum up support from software developers and interest from consumers.
PS2, or perhaps even both, I will assume that these titles do not influence a consumer’s decision to purchase a PS2. It seems reasonable that a consumer interested in playing older generation titles either would already own one or purchase the much cheaper console, and that the role of these new PS1 titles on the decision to purchase a PS2 is marginal at best.

Thirdly, the PS2 exhibited shortages during the first few months of its launch and supply was not able to meet demand, and without correction the model would potentially predict a lower value for \( \delta_{i,j,t} \) than the true value for those early months. However, if I assume that during these months access to the console was independent of consumer heterogeneity (and consumers purchased in the same proportion had there not been a shortage), then ignoring the implied \( \delta_{i,j,t} \) for the first few months during estimation would still yield consistent estimates.\(^{88}\) Eliminating observations from the first few months did not significantly affect results.

### A.3 Computational Details

Let \( i \in \mathbb{I} \equiv \{0, 1\}^3 \) denote the inventory state of a consumer. Slightly abusing notation, \( i = 0 \) will indicate no platforms are owned; \( i = 7 \) that all 3 have been purchased. I discretize the distributions of \( \alpha^p \) and \( \alpha^\gamma \) to model consumer heterogeneity: consumers can be divided among \( R \) groups (indexed by \( i \), each with price-sensitivity and gaming-preference coefficients \( (\alpha_i^p, \alpha_i^\gamma) \)) and initial population share \( \lambda_{i,t=0,i=0} \) according to the distributional assumptions given. At each period, the fraction of each type of consumer on a given console is given by \( \lambda_{i,t,i}^j \). For estimation, \( \alpha^\gamma \) takes on 11 distinct values, and \( \alpha^p \) has 5, resulting in \( R = 55 \) distinct consumer types.

The estimation routine has the following steps:

- To evaluate a candidate \( \theta \), iterate on the following until convergence on \( \{\Gamma_{j,t}(\alpha_i^p, \alpha_i^\gamma, \nu^p_{i,j,k}; t)\}_j \in J, \forall t,i \) is obtained:

  - **Hardware Adoption:**
    
    For a given \( \{\Gamma_{j,t}(\alpha_i^p, \alpha_i^\gamma, \nu^p_{i,j,k}; t)\}_j \in J, \forall t,i \) determine mean console utilities \( \{\delta_{i,j,t}(\cdot)\}_j \in J, \forall t \) which match observed shares in data with those predicted by the model. Also obtain distribution of consumer types who have purchased a given console at any period of time.

  - **Software Adoption:**
    
    Given the distribution of consumers onboard any hardware platform, compute mean software utilities \( \{\xi_{j,k,t}(\cdot)\}_j \in J, \forall k \in K_{j,t} \) for every software title on every platform that, again, match observed shares in data with those predicted by the model. Update implied software utilities \( \{\Gamma_{j,t}(\alpha_i^p, \alpha_i^\gamma, \nu^p_{i,j,k}; t)\}_j \in J, \forall t,i \).

- **Computation of Likelihood:** Obtain

  \[
  \{\nu^p_{j,t}\}_j \in J, \forall t = \{\xi_{j,t}(\theta) - \rho^p h_j \xi_{j,t-1}(\theta)\}_j \in J, \forall t
  \]

  \[
  \{\nu^w_{j,k,t}\}_j \in J, \forall k \in K_{j,t} = \{\eta_{j,k,t}(\theta) - \rho^w \eta_{j,k,t-1}(\theta)\}_j \in J, \forall k \in K_{j,t}, \forall t
  \]

  from recovered mean-utilities \( \{\delta_{i,j,t}\}_j \in J, \forall t \) and \( \{\xi_{j,k,t}\}_j \in J, \forall k, \forall \theta \), and form likelihood \( L(\theta) \).

Details are as follows.

#### Hardware Adoption

Note that the values \( \{\delta_{i,j,t}(\cdot)\}_j \in J \) are sufficient to compute the expected probabilities that a consumer with inventory \( i \), prior to realizing \( \epsilon_{i,t} \), will purchase any hardware console at time \( t \):

\[
\hat{\delta}_{i,t}(\{\delta_{i,j,t}(\cdot)\}_j \in J \in \forall t, m(t, \beta) = \frac{\exp(\delta_{i,t}(\cdot))}{\exp(\delta_{i,t}(\cdot)) + \exp(\beta E[V_t(\{\delta_{i,j,t+1}(\cdot)\}_j \in J, m(t+1)))]\{\delta_{i,j,t}(\cdot)\}_j \in J, \forall t, m(t)} \tag{26}
\]

as well as the probability a consumer purchases a particular hardware platform \( j \) conditional on purchasing any platform:

\[
\hat{\delta}_{i,j,t}(\{\delta_{i,j,t}(\cdot)\}_j \in J) = \frac{\exp(\delta_{i,j,t}(\cdot))}{\exp(\delta_{i,t}(\cdot))} \tag{27}
\]

where \( \hat{\delta}_{i,t}(\cdot) = \ln(\sum_{j \in J} \exp(\delta_{i,j,t}(\cdot))) \). (These are the standard “logit” closed form expressions derived from integrating over the extreme value errors \( \epsilon_{i,t} \). Thus, provided the values of \( \{\delta_{i,j,t}(\cdot)\}_j \in J, \forall t \), for each consumer \( i \), aggregation over (26) and (27) yields the total predicted share of consumers (who have not yet purchased a console) that purchases console \( j \) at time \( t \):

\[
\hat{s}_{j,t}(\{\delta_{i,j,t}(\cdot)\}_j \in J, \forall t, m(t, \beta, \alpha, \Sigma) = \sum_{i \in I} \int_{\alpha^p, \alpha^\gamma} \hat{\delta}_{i,t}(\cdot) \hat{\delta}_{i,j,t}(\cdot) dP_{i}(\alpha^p, \alpha^\gamma) \tag{28}
\]

\(^{88}\)This is equivalent to removing the initial values of \( \nu^p_{j,t} \) for the PS2 from the likelihood function.
where \( dP_i(\cdot) \) represents the distribution of consumer types who have not yet purchased a hardware system at time \( t \), and consumer heterogeneity is parameterized by mean \( \alpha \) and variance \( \Sigma \). The distribution of remaining consumers \( dP_i \) changes over time according to the population of consumers who have purchased in previous periods; this is one of the primary ways the demand system generates interdependence over time.

Define the mean utility for the “mean” consumer \((t = 0)\) of hardware platform \( j \) at time \( t \) and inventory state \( \iota = 0 \) as

\[
\delta_{j,t} \equiv \alpha^x x_{j,t} - \alpha^h p_{j,t} \cdot p_{j,t} + \Gamma_{j,t}(\alpha_0^x, \alpha_0^h; \iota = 0) + \zeta_{j,t} \tag{29}
\]

For a fixed \( \theta_1 \), \( \{\Gamma_{j,t}(\alpha_0^x, \alpha_0^h; t)\}_{j \in J_{t-1, t, i, i}} \), and \( \{\lambda_{i,0,\iota}\}_{\iota \in I_{t-1, t, i, i}} \), the contraction mapping introduced in Berry, Levinsohn, and Pakes (1995)

\[
\delta_{j,t}^{n+1} \equiv \delta_{j,t}^{n} + \psi \left( \ln(s^\iota j,t) - \ln(\hat{s}_{j,t}(\delta_{j,t}^{n-1})) \right) \tag{30}
\]

is used to obtain meaningful approximations \( \delta_{j,t}(\cdot) \), where \( s^\iota \) denotes the observed share of potential buyers who purchase console \( j \) at time \( t \) and \( \psi \in (0, 1) \) is a “tuning” parameter. For the following discussion, I will omit arguments \( \Gamma, \beta, \theta, \) and \( m(t) \) whenever it is possible to do so without confusion.

- At each stage of the mapping, implied market shares \( \hat{s}_{j,t}(\delta_{j,t}^{n-1}) \) are computed according to:

\[
\hat{s}_{j,t}(\delta_{j,t}^{n-1}) = \sum_{i=1}^{6} \sum_{\iota=1}^{R} \lambda_{i,\iota,t} \hat{s}_{i,\iota,t} (\delta_{i,\iota,t}; \theta) \hat{s}_{i,j,\iota|t} \tag{31}
\]

- To obtain \( \hat{s}_{i,\iota,t} (\delta_{i,\iota,t}) \) and \( \hat{s}_{i,j,\iota|t} \), the consumer dynamic optimization problem for a given \( \delta_{j,t}^{n-1} \) and for every consumer type \( i \) and inventory state \( \iota \) must be solved. Noting \( \delta_{i,\iota,t} (\iota) = \delta_{i,\iota,t} - (\alpha_0^x - \alpha_0^h) p_{j,t} + \Gamma_{j,t}(\alpha_0^x, \alpha_0^h; \iota = 0) \), the transition kernel according to the regression in (19) is first updated. I assume there is a finite horizon \( T \) at which point \( \{\delta_{j,t}(\cdot)\}_{\iota \in I_{t-1, t, i, i}} \) decays to 0, and simulate forward 50 sample paths to compute the expected value function \( \{EV_i(\delta_{i,j,t}(\iota))\}_{\iota \in I_{t-1, t, i, i}} \) given by (25).\(^8^9\)

In practice, I assume this horizon occurs in January 2006, 3 months after the end of the data sample; however, results did not change significantly when the horizon was extended by 1 year.

- To update \( \{\lambda_{i,0,\iota}\}_{\iota \in I_{t-1, t, i, i}} \), first shares \( \{\lambda_{i,t=0,\iota=0}\}_{\iota=0} \) are computed from the distribution implied by \( \theta_1 \), and then each future period is computed by updating the distribution of consumers remaining on the market as follows:

\[
\lambda_{i,t+1,\iota} = \frac{(1 - \hat{s}_{i,t,\iota}) \lambda_{i,t,\iota} + \sum_{\iota' \in I^- (\iota)} \lambda_{i,t,\iota'} \hat{s}_{i,t,\iota'} \hat{s}_{i,\iota', t | \iota'} \iota' | t}{\sum_{\iota'' \in I^- (\iota)} \lambda_{i,t+1,\iota''}} \tag{32}
\]

where \( I^- (\iota) \) is the set of inventory states that can “reach” inventory state \( \iota \) — e.g., differ only by having one fewer console. In other words, the share of consumers with inventory \( \iota \) at time \( t + 1 \) is simply those that did not purchase a new console at time \( t \) (the first term of the numerator) plus those in state \( \iota' \) at time \( t \) who purchase console \( j \), where \( j \) is the only difference between \( \iota \) and \( \iota' \). To account for the growth in total market size (i.e., more television households are present in each period), I assume that new households are distributed across consumer types according to their initial distribution.

### Software Adoption

As on the hardware side, note the mean utility for consumer \( i \), \( \zeta_{i,k,t} \), is a sufficient statistic in determining whether or not consumer \( i \) purchases software title \( k \) in a given period. The model implies that the share of people who purchase title \( k \) is given by:

\[
s_{j,k,t} = \int \frac{\exp(\zeta_{i,k,t})}{\exp(\zeta_{i,k,t}) + \exp(\beta E[\zeta_{i,k,t+1}|\zeta_{i,k,t}])} dP_{j,k,t}(\alpha^x, \alpha^y) \tag{33}
\]

\(^8^9\)I also explored using a discretized state space with a non-uniform grid (concentrating points in areas that are more likely to be visited), simplical interpolation, and standard value function iteration for convergence. Even when using a relatively sparse grid of 20 points in each direction for the \( \delta_{j,t} \) terms, the state space is of size \( 8 \times 12 \times 20^3 = 8 \times 10^7 \). Initial results were similar to those obtained using a finite horizon, but this method did not scale well when the number of consumer types increased and as a result became too computationally unwieldy. Attempts at using polynomial approximations with shape-preserving splines were also unsatisfactory; similar to the issue raised in Section 9 of Benitez-Silva, Hall, Hitsch, Pauletto, and Rust (2000), polynomial approximation routines do not capture the location of the kink in the value function of a consumer’s optimization problem accurately. As this value determines whether or not the consumer purchases, it is of central importance in computing the likelihood function, and thus these inaccuracies render the approximations unsuitable despite their computational advantages.
where $P_{j,k,t}(\cdot)$ is distribution of consumer characteristics for those who own platform $j$ but have not bought software $k$ at time $t$; it is a function of the hardware adoption decision for all consumers in periods $\tau \leq t$. It is the evolution of this distribution over time and across platforms that necessitates the joint estimation of software and hardware demand.

To obtain a starting value of $\{\Gamma_{j,t}(\cdot);t\}_v$, for the hardware adoption side, I first assume $\{\lambda_{j,t}^i\}_v = \lambda_{j,0}^i$ - i.e., the entry distribution of consumer types on each hardware platform is stationary across time. For a given $\theta_1$, $\{\lambda_{j,t}^i\}_{v,j\in J_j,t}$, the software side proceeds in a parallel fashion to the hardware adoption side. For each console $j$, the same BLP contraction mapping is used to recover mean software qualities $\zeta_{j,k,t}$:

$$
\zeta_{j,k,t}^n(\theta_1, \{\lambda_{j,t}^i\}) = \zeta_{j,k,t}^{n-1} + \ln(s_{j,k,t}) - \ln(s_{j,k,t}(\zeta_{j,k,t}^{n-1}))
$$

- Implied market shares $\hat{s}_{j,k,t}(\zeta_{j,k,t}^{n-1})$ are computed as in the hardware side (except now there are only two inventory states $\{0,1\}$), where the initial base of consumers who have not purchased a title is given by the demand of consumers on a given console at the time of the title’s release, and each future period’s potential market size is updated accordingly. Again, the consumer dynamic optimization problem given by (11) for a given $\zeta_{n-1}^{j,k,t}$ is solved for every consumer type $i,k,t$. I discretize the state space into a uniform grid with $201 \times 12$ points, and employ Halton sequences for random draws on the evolution of $\zeta_{i,j,k,t}$, simple linear interpolation, and standard value function iteration for convergence. At each stage, transition kernel is updated according to the regression in (20).

Once the expected value function is computed for each software title, consumer type, and time period, $\bar{\Gamma}$ is updated according to (11).

**Recovery of $\xi(\theta)$ and $\eta(\theta)$**

The hardware and software adoption algorithms are repeated until convergence on $\{\Gamma_{j,t}\}_{j\in J_j,v,i}$ and $\{\lambda_{j,t}^i\}_{v,j\in J_j,t}$ is obtained. This yields $\{\hat{\xi}_{j,t}(\theta_1)\}_{j\in J_j,v,i}$ and $\{\hat{\eta}_{j,k,t}(\theta_1)\}_{j\in J_j,k,t}$. Note:

$\hat{\xi}_{j,t}(\theta_1) = \delta_{j,t}(\theta_1) + \alpha_0^{p,hw}p_{j,t} - \Gamma_{j,t}(\alpha_0^{p,hw};t = 0) - \delta_{j,t}w_{j,t}$

$\hat{\eta}_{j,k,t}(\theta_1) = \zeta_{j,k,t}(\theta_1) + \alpha_0^{p,sw}p_{j,k,t} - \alpha^{w}w_{j,k,t}$

Consequently, given assumption 3.6, $\nu_{j,t}^{hw}(\theta_1)$ can be recovered as the residual from the OLS regression of $(\delta_{j,t}(\theta_1) + \alpha_0^{p,hw}p_{j,t} - \Gamma_{j,t}(\alpha_0^{p,hw};t = 0) - \delta_{j,t}w_{j,t})$ on $(x_{j,t} - \rho_{j,t}w_{j,t})$. Similarly, $\nu_{j,k,t}^{sw}(\theta_1)$ can be recovered from the partial regression of $(\zeta_{j,k,t}(\theta_1) + \alpha_0^{p,sw}p_{j,k,t} - \rho_{j,t}w_{j,k,t})$ on $(w_{j,k,t} - \rho_{j,t}w_{j,k,t})$. The use of MLE and normally distributed errors allows for recovering $\theta_2 \equiv \{\alpha^{p}, \alpha^{w}\}$ as a linear function of $\theta_1$ in this manner; i.e., $\theta_2$ can be “concentrated” out at this stage and a non-linear search is conducted only over $\theta_1$. To estimate software-title fixed effects contained within $\alpha^{w}$ (of which there are over a thousand), a partitioned regression is used.

Finally, the log-likelihood provided by (16) can be evaluated from these residuals.

**A.4 Fit of Model**

Figure 4 plots the predicted values of $\{\delta_{j,t}\}_{j\in J_j,v;i}$ for the mean consumer. For this and all following figures, I use the results from the full demand model given by specification (v) in table 3. Consumers’ expectations of hardware mean-utilities $\{\delta_{j,t}\}_{j\in J_j,v;i}$ are assumed to be a function of the previous values across all consoles as well as the time of year: as evident, values for each console do seem to track each other, and each is drastically affected by seasonality with a large increase occurring during the holiday seasons. To determine whether conditioning only on these past variables, as given by the parameterization in (19), provides a reasonable approximation of consumer expectations, figure 5 plots the error in the realized value of $\delta_{j,t+1}$ from the expected value implied by the estimated transition process $F_t(\{\delta_{j,t}\}_{j\in J_j,m(t)})$. For the most part, these predicted errors are relatively small, with their variance on average less than 15% of the variance in the actual change in $\delta_{j,t+1} - \delta_{j,t}$. There does not seem to be any persistent correlation or time trends. I also computed these errors without explicitly accounting for seasonality effects, and errors were substantially larger in magnitude - nearly quadrupling in variance - indicating once again the importance of controlling for the time of year.

From the demand estimates, there is significant persistence in hardware and software unobservables with $\rho_{hw}$ estimated to be .65 and $\rho_{sw}$ to be .81. A value of $\rho_{hw}$ at .65 indicates the degree of variance in $\xi_{j,t}$ explained by $\xi_{j,t-1}$ is .65$^2 \approx 42\%$, which indicates there is relatively a large amount of unexplained variation across periods in the hardware unobservable characteristics. However, this variation in $\xi$ given by $\nu_j^{hw}$ only comprises $10 - 15\%$ of the total variance in $\delta_{j,t}$ across consoles, and thus does not necessarily indicate lack of explanatory power on the part of the model.
The key assumption used for inference is 3.6, which states the evolution of $\nu^{bw}$ and $\nu^{sw}$ in unobserved product characteristics $\xi$ and $\eta$ is independent of each other and the change in observed characteristics at each point in time. Figure 6 plots the implied values of $\nu^{bw}$ for each hardware device. These values also appear to be uncorrelated across hardware platforms and across time, something statistical tests cannot reject. Thus, there do not seem to be any significant common shocks across platforms to hardware unobservable characteristics that are not already accounted for by changes in the observed characteristics $x_{j,t}$.

I next examine the predicted number of consumers who multihome and purchase multiple consoles. Although the data indicates 53.2M sixth-gen consoles were sold, the model predicts 47M households actually purchased one or more consoles—i.e., 21% of households are predicted to purchase more than one console, in which only a very small number (<1%) purchase all 3. Nielsen Media estimates that at the end of 2005, there were 43M households in the US that owned a videogame console.\footnote{\textit{The State of the Console}, The Nielsen Company, 2007.} Although the model slightly underestimates the amount of multihoming if the Nielsen data is accurate, it does provide substantial correction for the possibility consumers purchase multiple devices. The model also indicates that different consoles exhibit different propensities for multihoming: whereas 15% and 24% percent of PS2 and Xbox owners are predicted to own more than 1 console, over 32% of GC owners do so.

### A.5 Alternate Formulation for Future Software Utility

This section details an alternative specification for future software utility, $\Lambda_{j,t}(\cdot)$. I assume that consumers form expectations over future software availability in a two-step procedure. First, every consumer has the same expectations over the number of titles released in each month, which is itself a function only of the number of current software titles available and the time period (for age and month effects); secondly, each consumer perceives the expected mean utility of each new title $\zeta_{i,k,t}, k \in K_{j,R(t+1)}$ is independently drawn from a distribution that is consistent with the observed $\zeta$’s of titles released in a given month.\footnote{This can easily be extended to allow consumers to have adaptive expectations; e.g., expected software utilities are drawn from the observed distribution of only those titles released prior to time $t$.}

#### Assumption A.1

Let $q_{j,t} \equiv |K_{j,R(t)}|$, the number of new software titles released for platform $j$ at time $t$, and let $Q_{j,t} \equiv |K_{j,t}|$ be the number of total software titles currently available (i.e., released during and before period $t$). Consumers perceive the distribution of $q_{j,t}$ is a function only of $Q_{j,t}$, $m(t)$, and the platform $j$’s age $a_{j,t}$:

$$H(q_{j,t+1}|\Omega_j^t) = H_j(q_{j,t+1}; Q_{j,t}, m(t+1), a_{j,t+1})$$

Additionally, the probability a future title has quality $\zeta_{i,k,t}$ is a function only of the platform and month in which the title will be released:

$$J(\zeta_{i,k,t+\tau}|\Omega_j) = J_{i,j}(\zeta_{i,k,t+\tau}|m(t+\tau)) \forall k \in K_{j,R(t+\tau)}$$

Clearly, the number of titles released in a future period depends on more factors than just the number of titles currently available, most notably the installed base of a given console. Nonetheless, there are two reasons why assumption A.1 may not be problematic: (i) the total number of software titles currently available itself is a proxy for the proclivity of further software development as it should in turn reflect the size of the installed base;\footnote{Simple tests reject the hypothesis that installed base and software release numbers do not Granger-cause the other.} (ii) most consumers do not have accurate information as to a console’s current installed base, but do know the number of titles currently available (given by simply looking at store shelves).

A consumer thus perceives future software utility of a platform as a series of iterated expectations over both the number ($q_{j,t}$) and quality ($\zeta_{i,k,t}$) of software titles to be released in all future periods:

$$\Lambda_{j,t}(\alpha_t^\zeta, \alpha_t^\mu) = E_j \left[ \sum_{k=1}^{q_{j,t+1}} \zeta_{i,k,t+1} + E_{t+1} \left[ \sum_{k=1}^{q_{j,t+2}} \zeta_{i,k,t} + E_{t+2}[\cdots]|Q_{j,t}, q_{j,t+1}|Q_{j,t} \right] \right]$$

Computation: Before the main routine is begun, I first estimate “offline” consumers’ beliefs over the number of software titles released in future months, which is given by the distribution function $H(\cdot)$ in assumption A.1. A natural candidate would be to use a poison distribution. However, since there is overdispersion in the data, a negative binomial fixed effects model similar to that proposed in Hausman, Hall, and Griliches (1984) is more suitable. I thus assume $h(\cdot)$ has the density of a negative binomial, but of a specification slightly different than theirs.\footnote{Cameron and Trivedi (1998) refer to the specification used by Hausman, Hall, and Griliches (1984) as an NB1 approximation.} The mean of the negative binomial is $\mu$ and the variance is $\mu + \alpha^2\mu$. I parameterize the conditional mean as $\ln(\mu) = \beta^\mu x^\mu_{j,t}$, where $x^\mu_{j,t}$ contains both $Q_{j,t}$ as well month dummies and platform age terms, and the dispersion term is parameterized
as \( \ln(\alpha) = x_j^o \beta_\alpha \), where \( x_j^o \) are platform dummies. For estimation, I use the number of titles released for platforms which were released in the sixth as well as fifth-generation, and extend out the sample to April, 2007.\(^4\) Once this model is estimated, there is still the issue of forming consumer expectations at time \( t \) over the path of software titles in all future periods. This integral is intractable, but can be approximated via forward simulation. For each console and every \( t \), I first take a draw on \( q_{j,t+1} \) conditional on observed \( Q_{j,t} \), and then proceed forward by updating \( Q_{j,t+1} = Q_{j,t} + q_{j,t+1} \), where \( q_{j,t+1} \) is the value previously drawn. Updating to a terminal value and creating multiple sample paths allows for the approximation of \( \{E_x[(q_{j,t+\tau})_{\tau>0}, \gamma_j]\}_{\tau=t} \).

Table 13 presents the results of the negative binomial regression on the number of software titles released in the next month as a function of the total number of titles currently available and age of the console. The more titles currently available indicates the higher likelihood of titles being released in the future, whereas age negatively affects this probability. Seasonality plays a significant role, with most titles likely to be released in the months prior to the holiday season. Finally, there is overdispersion predicted in the data, and \( \alpha \) is predicted to be the largest for the PS2, which had the most titles released in the sample, and the smallest for the GC, which had the least number of titles.

One question is the fit of this model in predicting the future number of titles released. Figure 7 graphs the number of titles released for each console the 5% and 95% percentile of the predicted negative binomial distribution. With only a few exceptions, the number of titles released for all three consoles fall within the interval. The larger variance of titles released during the peak months and the differing variances across consoles is captured by the model. Though only the dispersion fixed-effects for the sixth generation consoles are reported, including the data from the previous generation of consoles helped match the observed frequency of software releases to the confidence interval predicted by the model.

Using this alternate formulation for demand estimation yielded much lower estimates of \( \beta^* \) – i.e., a greater discounting of future software – but other parameters of the model did not significantly change.\(^9\) As a result, the substantive predictions from the demand model were not significantly affected.

## B Software Network Formation Game: Computation of Profits

To describe how expected profits are computed, I rewrite (21) as:

\[
E[\pi_k(s_k; \theta_C) | \Omega_{r_k-\tau}] = \left( \sum_{t=r_k}^{T} \beta^{t-r_k} \sum_{j \in s_k} E[M_{j,k,t} s_{j,k,t} ((1 - rmkup_j) p_{j,k,t} - mc_j)] \right) - C_k(s_k; \theta_C)
\]

where now \( Q_{j,k,t} \) has been broken into two different components: \( s_{j,k,t} \), which represents the share of consumers who purchase title \( k \), and \( M_{j,k,t} \), which represents the number of consumers on platform \( j \) who have not yet purchased title \( k \). \( s_{j,k,t} \) is defined in (33), and is solely a function of \( Q_{k,t} \), and the distribution of consumer types onboard platform \( j \) who have not yet purchased the title. If \( IB_{j,t} \) is the number of consumers who own console \( j \) at time \( t \), and \( IB_{j,k,t} \) is the number of consumers who own title \( k \) on platform \( j \), then \( M_{j,k,t} = IB_{j,t} - IB_{j,k,t} \), where \( IB_{j,k,t} = IB_{j,k,t}^{s_{j,k,t}} + M_{j,k,t-1} s_{j,k,t-1} \). From the demand side, recall a sufficient statistic for determining \( IB_{j,t} \) is \( IB_{j,t-1} \) and \( \{\delta_{j,t}\}_{j \in J_k} \).\(^96\)

To form the first part of \( E[\pi_k(s_k; \theta_C)] \), only expected values of \( \{\delta_{j,t}, s_{j,k,t}, p_{j,k,t}\}_{j \in J_k, t > r_k - \tau} \) are first required. I obtain these using a simulated frequency approach as in Pakes (1986): multiple sample paths of these variables are created via forward simulation using the estimated transition processes from the demand system (given by (19), (20), and table 14), and the appropriate quantities \( M_{j,k,t} \) and \( s_{j,k,t} \) are calculated at each point in time, again from the demand system. At release date \( r_k \), the predicted hardware mean utilities \( \{\delta_{j,t}\} \) are increased by the amount software \( k \) contributes to each platform it joins, as determined by its choice of strategy \( s_k \).

model, whereas I use what they refer to as the NB2 specification:

\[
h(q|\mu, \alpha) = \frac{\Gamma(q + \alpha^{-1})}{\Gamma(q + 1) \Gamma(\alpha^{-1})} \left( \frac{\alpha^{-1}}{\alpha^{-1} + \mu} \right)^{\alpha^{-1}} \left( \frac{\mu}{\alpha^{-1} + \mu} \right)^q
\]

where \( \Gamma \) is the gamma function (not to be confused with the definition of software utility \( \Gamma(.)_{j,t} \) used in this paper): \( \Gamma(\alpha) = \int_0^\infty e^{-t} t^{\alpha-1} dt \), \( \alpha > 0 \)

\(^94\)The number of software titles released for each of the three sixth-generation consoles from November 2005 to April 2007 was collected from http://www.gamespot.com, an online videogame website.

\(^95\)This may indicate this particular construction of future software utility tends to overpredict the supply of “hit” titles, and drawing from the distribution of software quality released in each period may not be the best approximation to software supply.

\(^96\)Although for notational simplicity I have omitted discussing the distribution of consumer heterogeneity and inventory states, these concerns are not ignored in estimation.

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To simulate each title’s expected price path, I assume each software title perceives it to also follow a first-order Markov process, and depend only on its own previous value. Table 14 provides an OLS regression of software prices on lagged prices, and shows that previous prices explain over 90% of the variation in next period prices. Additionally, although the number of copies sold in the previous period is also a significant factor in determining next period’s prices, its inclusion in simulating price paths did not significantly affect results; this is unsurprising given the limited improvement in the $R^2$ of the initial regression.\(^{97}\)

C Proofs

C.1 Proof of Proposition 3.7

The proof follows closely Chanda (1954) and Bradly and Gart (1962), and provides necessary modifications where needed.\(^{98}\) The assumptions required for the proof are as follows:

Assumption C.1. Let $\theta = (\theta_1, \ldots, \theta_k)$ and $i \in \{hw, sw\}$.

i. For almost all $x \in \mathbb{R}$ and for all $\theta \in \Theta$

$$\frac{\partial \ln f^i}{\partial \theta_r}, \frac{\partial^2 \ln f^i}{\partial \theta_r \partial \theta_s} \text{ and } \frac{\partial^3 \ln f^i}{\partial \theta_r \partial \theta_s \partial \theta_t} \quad r, s, t = 1, \ldots, k$$

exist.

ii. For almost all $x \in \mathbb{R}$ and for all $\theta \in \Theta$,

$$\left| \frac{\partial f^i}{\partial \theta_r} \right| < F_{ir}(x), \quad \left| \frac{\partial^2 f^i}{\partial \theta_r \partial \theta_s} \right| < F_{irs}(x) \quad \text{and} \quad \left| \frac{\partial^3 f^i}{\partial \theta_r \partial \theta_s \partial \theta_t} \right| < H_{irrst} \quad r, s, t = 1, \ldots, k$$

where $F_{ir}(x)$ and $F_{irs}(x)$ are integrable over $\mathbb{R}$, and $E[H_{irrst}(x)] < M^1$, where $M^1$ are finite positive constants.

iii. For all $\theta \in \Theta$, the matrix $J = [J_{rs}(\theta)]$ with

$$J_{rs}(\theta) = \left[ \sum_{j=1}^J \mu_{j}^{hw} \int \left( \frac{\partial \ln f^{hw}_j}{\partial \theta_r} \frac{\partial \ln f^{hw}_j}{\partial \theta_s} \right) f^{hw}_j dx + \mu_{j}^{sw} \int \left( \frac{\partial \ln f^{sw}_j}{\partial \theta_r} \frac{\partial \ln f^{sw}_j}{\partial \theta_s} \right) f^{sw}_j dx \right]$$

is positive definite with finite determinant.

These are different from, and in a sense stronger than, those used in Wald (1949) for his general proof on the consistency of MLE. The assumptions here, however, are easier to check and confirm.

Recall the log-likelihood function to be maximized is

$$\mathcal{L}(\theta) = \sum_{j=1}^J \sum_{t=K_j+1}^{K_j+K_{j,t}} \left( f^{hw}_j (\nu_j^{hw}_j(\theta); \theta) + f^{sw}_j (\nu_j^{sw}_j(\theta); \theta) \right)$$

and $n_{j}^{hw} = \sum_{t=K_j+1}^{K_j+K_{j,t}} 1$ and $n_{j}^{sw} = \sum_{t=K_j+1}^{K_j+K_{j,t}} \sum_{k=1}^{K_{j,t}} 1$ be the number of observations for platform $j$, and $N = \sum_{j=1}^J (n_{j}^{hw} + n_{j}^{sw})$ be the total number of error observations. Let $\theta^0$ represent the true value of $\theta$.

Consistency: Let $\theta^0 \in \Theta$ be the true value of the parameter vector $\theta$ to be estimated. Take the following Taylor expansion for $i \in \{hw, sw\}$:

$$\frac{\partial \ln f^i}{\partial \theta_r} = \left( \frac{\partial \ln f^i}{\partial \theta_r} \right)_{\theta = \theta^0} + \sum_{s=1}^k (\theta_s - \theta^0_s) \left( \frac{\partial^2 \ln f^i}{\partial \theta_r \partial \theta_s} \right)_{\theta = \theta^0} + \frac{1}{2} \sum_{s, t=1}^k (\theta_s - \theta^0_s)(\theta_t - \theta^0_t) \left( \frac{\partial^3 \ln f^i}{\partial \theta_r \partial \theta_s \partial \theta_t} \right)_{\theta = \theta^0} $$

\(^{97}\)Note that $\zeta$ includes price, and yet is assumed to evolve in an independent process from price itself. To address this concern, I also estimate an alternative specification in which I assume software mean quality net of price (i.e., $\zeta_{k,t} = \zeta_{k,t} + \alpha_{p,k,t}^i p_{k,t}$) evolves according to a Markov process, and proceed in a similar fashion (where $\zeta_{k,t}$ is constructed in each period from the separate evolution in $\zeta_{k,t}$ and $p_{k,t}$). Results do not change in any significant way.

\(^{98}\)NB: there is an error in Chanda (1954)’s proof of uniqueness which is addressed and fixed in Tarone and Gruenhage (1975) (which also has a corrigenda in 1979).
where \( \theta' = \theta'(x) \) is a value that depends on \( x \) but for all \( x \) lies within the hyper-cell containing \( \theta - \theta^0 \) as its diagonal. Summing these Taylor expansions across \( i, j, t, k \) and dividing by \( N \) yields:

\[
\frac{1}{N} \frac{\partial \mathcal{L}}{\partial \theta_r} \equiv L_r(\theta) = L_r(\theta^0) - \sum_{s=1}^{k} \delta_s L_{rs}(\theta^0) + \frac{1}{2} \sum_{s,t=1}^{k} \delta_s \delta_t L_{rs,t}
\]

(34)

where

\[
\delta_a = \theta_a - \theta_a^0
\]

\[
L_r(\theta) = \sum_{j=1}^{j} \left( \mu_j^{hw} \sum_{t=r_j}^{T} \frac{1}{n_j^{hw}} \frac{\partial \ln f_{r_j,t}^{hw}}{\partial \theta_r} + \mu_j^{sw} \sum_{t=r_j}^{T} \frac{1}{n_j^{sw}} \frac{\partial \ln f_{r_j,t}^{sw}}{\partial \theta_r} \right)
\]

\[
L_{rs}(\theta) = \sum_{j=1}^{j} \left( \mu_j^{hw} \sum_{t=r_j}^{T} (-1) \frac{1}{n_j^{hw}} \frac{\partial^2 \ln f_{r_j,t}^{hw}}{\partial \theta_s \partial \theta_r} + \mu_j^{sw} \sum_{t=r_j}^{T} \sum_{k=1}^{K_{j,t}} (-1) \frac{1}{n_j^{sw}} \frac{\partial^2 \ln f_{r_j,t}^{sw}}{\partial \theta_s \partial \theta_r} \right)
\]

\[
L_{rs,q}(\theta) = \sum_{j=1}^{j} \left( \mu_j^{hw} \sum_{t=r_j}^{T} \frac{1}{n_j^{hw}} \frac{\partial^3 \ln f_{r_j,t}^{hw}}{\partial \theta_q \partial \theta_s \partial \theta_r} \right) + \mu_j^{sw} \sum_{t=r_j}^{T} \sum_{k=1}^{K_{j,t}} \frac{1}{n_j^{sw}} \frac{\partial^3 \ln f_{r_j,t}^{sw}}{\partial \theta_q \partial \theta_s \partial \theta_r} \right)
\]

From C.1 it follows that

\[
L_r(\theta^0) \rightarrow_p 0
\]

\[
L_{rs}(\theta^0) \rightarrow_p J_{rs}(\theta^0)
\]

\[
|L_{rs,q}| < \sum_{j=1}^{j} \mu_j^{hw} M^{hw} + \mu_j^{sw} M^{sw}
\]

With both \( f^{hw} \) and \( f^{sw} \) to consider, both Khintchine’s Theorem (Rao (1973), 2.c.ii) and Slutsky’s Theorem are required for convergence results. E.g., since \( E[\partial \ln f_{r_j,t}^{hw} / \partial \theta_r] = 0 \), Khintchine’s Theorem proves \( \sum_{t=r_j}^{T} (1/n_j^{hw}) (\partial \ln f_{r_j,t}^{hw} / \partial \theta_r) \rightarrow_p 0 \); Slutsky’s Theorem handles the sum of two convergent sequences and leads to the result \( L_r(\theta) \rightarrow_p 0 \). The same arguments apply for the last two results.

The remainder of the consistency proof follows Chanda (1954) using the same notation, and is omitted here. **Asymptotic Normality:** Following Bradley and Gart (1962), substitute \( \hat{\theta} \) into (34), note \( L_r((\theta)) = 0 \), and rearrange to obtain:

\[
L_r(\theta^0) = \sum_{s=1}^{k} (\hat{\theta}_s - \theta_s^0) L_{rs}(\theta^0) - \frac{1}{2} \sum_{s,t=1}^{k} (\hat{\theta}_s - \theta_s^0)(\hat{\theta}_t - \theta_t^0) L_{rs,t}
\]

for \( r = 1, \ldots, k \), or in matrix notation:

\[
(L + G)(\hat{\theta} - \theta^0) = \left[ \sum_{j=1}^{j} \left( \mu_j^{hw} \sum_{t=r_j}^{T} \frac{1}{n_j^{hw}} \frac{\partial \ln f_{r_j,t}^{hw}}{\partial \theta_r} \right) + \mu_j^{sw} \sum_{t=r_j}^{T} \sum_{k=1}^{K_{j,t}} \frac{1}{n_j^{sw}} \frac{\partial \ln f_{r_j,t}^{sw}}{\partial \theta_r} \right] \quad (35)
\]

where \( L = [L_{rs}(\theta^0)] \) and \( G = [(-1/2) \sum_{s=1}^{k} (\theta_s - \theta_s^0) L_{rs,q}] \) are k-square symmetric matrices. In the proof of consistency, I have shown \( L \rightarrow_p J_0 \) (which is positive definite, by assumption), and also can show \( G \rightarrow_p 0 \) since \( \hat{\theta} \rightarrow_p \theta^0 \) and \( |L_{rs,q}| \) is bounded. Thus, for large \( T \), \( L + G \) may be inverted and by Slutsky’s Theorem, \( L + G \rightarrow_p J_0 \) and \( (L + G)^{-1} \rightarrow_p J_0^{-1} \). Rewriting (35) yields:

\[
\sqrt{N}(\hat{\theta} - \theta^0) = (L + G)^{-1} \left[ \sum_{j=1}^{j} \left( \mu_j^{hw} \sum_{t=r_j}^{T} \frac{1}{n_j^{hw}} \frac{\partial \ln f_{r_j,t}^{hw}}{\partial \theta_r} \right) + \mu_j^{sw} \sum_{t=r_j}^{T} \sum_{k=1}^{K_{j,t}} \frac{1}{n_j^{sw}} \frac{\partial \ln f_{r_j,t}^{sw}}{\partial \theta_r} \right] \quad (35)'
\]

where the last term on the right, by the multivariate central limit theorem (Rao (1973), 2.c.ix), tends to a multivariate normal distribution with mean zero and variance covariance \( J_0 \). It thus follows that the asymptotic distribution of \( \sqrt{N}(\hat{\theta} - \theta^0) \) is multivariate normal with mean zero and variance covariance matrix \( J_0^{-1}J_0J_0^{-1} = J_0^{-1} \) as \( T \rightarrow \infty \) and \( \{\mu_j^t\}_{t \in \{hw,sw\}, r_j \in J} \) constant.
<table>
<thead>
<tr>
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<th>PS2</th>
<th>XBOX</th>
<th>GC</th>
<th>ALL</th>
</tr>
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<td><strong>HARDWARE</strong></td>
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<td></td>
<td></td>
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<tr>
<td>Months Active</td>
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<td>48</td>
<td>48</td>
<td></td>
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<tr>
<td>Price</td>
<td></td>
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<td></td>
<td></td>
</tr>
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<td>Average</td>
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<td>$185.47</td>
<td>$127.79</td>
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<td>36.50</td>
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<td>199.85</td>
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<td>146.92</td>
<td>92.37</td>
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<td>0.08</td>
<td>0.04</td>
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<td>9.83</td>
<td>53.22</td>
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<td>0.16</td>
<td>0.16</td>
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<td>Total # Titles Released</td>
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<td>749</td>
<td>487</td>
<td>1581</td>
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<td>% Exclusive</td>
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<td>33.4</td>
<td>27.5</td>
<td>62.7</td>
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<tr>
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<td>8.4</td>
<td>9.2</td>
<td>13.3</td>
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<tr>
<td>% Also on PS2</td>
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<td>67.6</td>
<td>73.4</td>
<td></td>
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<tr>
<td>% Also on XB</td>
<td>40.9</td>
<td>56.3</td>
<td>47.4</td>
<td></td>
</tr>
<tr>
<td>% Also on GC</td>
<td>28.3</td>
<td>36.6</td>
<td>30.8</td>
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<tr>
<td>% On all 3 Consoles</td>
<td>21.6</td>
<td>33.5</td>
<td>51.5</td>
<td>15.9</td>
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<td># Titles Released Per Month</td>
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<td>Average</td>
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<td>Max</td>
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Notes: Summary statistics for the PS2 are for the 61-month period between October 2000 and October 2005; statistics for the other two consoles are for a 48-month period beginning on November 2001.
Table 2: Top 10 Videogame Titles for Each Platform by Quantity Sold

<table>
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<tr>
<th>Console</th>
<th>Title</th>
<th>Publisher</th>
<th>Release Date</th>
<th>Exclusive?</th>
<th>Quantity ('000s)</th>
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<td>PS2</td>
<td>GTA: Vice City</td>
<td>Take 2</td>
<td>Oct 2002</td>
<td>No</td>
<td>6,687</td>
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<td></td>
<td>GTA: San Andreas</td>
<td>Take 2</td>
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<td>No</td>
<td>5,797</td>
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<td></td>
<td>GTA 3</td>
<td>Take 2</td>
<td>Oct 2001</td>
<td>No</td>
<td>5,588</td>
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<td></td>
<td>Gran Turismo 3: A-Spec</td>
<td>Sony</td>
<td>Jul 2001</td>
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<td>Madden NFL 2004</td>
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<td>EA</td>
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<td>Need for Speed: UG</td>
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<td>Project Gotham Racing</td>
<td>Microsoft</td>
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<td>Dec 2001</td>
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<tr>
<td></td>
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<td></td>
<td>Zelda: Wind Waker</td>
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<td>Mario Kart: Double</td>
<td>Nintendo</td>
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<td>Nintendo</td>
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<td>Metroid Prime</td>
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<td>Sonic Adventures 2</td>
<td>Sega</td>
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<td>Pokemon Colliseum</td>
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<td></td>
<td>Mario Party 4</td>
<td>Nintendo</td>
<td>Oct 2002</td>
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Notes: Total quantity sold is for the 61-month period between October 2000 and October 2005. Although non-exclusive, the PS2 had a window of exclusivity for these three titles: GTA: Vice City and GTA: 3 were not released on the Xbox until 2003 (well after they had both become blockbusters for the PS2), whereas GTA: San Andreas, though initially developed for both Xbox and PS2, was not released for the Xbox until 6 months after the PS2 game’s release.
Table 3: Estimated Parameters of Demand System

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<th>Variable</th>
<th>Static Model</th>
<th>No Consumer Heterogeneity</th>
<th>Dynamic Model</th>
<th>Dynamic Model</th>
<th>Full Consumer Heterogeneity</th>
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<td></td>
<td>Singlehoming</td>
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<td>(ii)</td>
<td>(iii)</td>
<td>(iv)</td>
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<tr>
<td></td>
<td>Parameter</td>
<td>Standard Error</td>
<td>Parameter</td>
<td>Standard Error</td>
<td>Parameter</td>
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<td>0.192</td>
<td>1.435</td>
<td>0.170</td>
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<tr>
<td></td>
<td>$\sigma_\gamma$</td>
<td>0.937</td>
<td>0.034</td>
<td>0.931</td>
<td>0.045</td>
</tr>
<tr>
<td></td>
<td>$\beta_\gamma$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$D$</td>
<td>-1.187</td>
<td>0.212</td>
<td>-1.187</td>
<td>0.212</td>
</tr>
<tr>
<td>Implied $\alpha_0^{p,hw}$</td>
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<td>1.978</td>
<td>1.887</td>
<td>1.887</td>
<td>1.887</td>
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<tr>
<td>Hardware Parameters</td>
<td>$d_{PS2}$</td>
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<td>1.899</td>
<td>5.309</td>
<td>0.653</td>
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<tr>
<td></td>
<td>$d_{XBOX}$</td>
<td>-22.857</td>
<td>2.857</td>
<td>0.545</td>
<td>0.603</td>
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<tr>
<td></td>
<td>$d_{GC}$</td>
<td>-19.785</td>
<td>2.896</td>
<td>0.586</td>
<td>0.595</td>
</tr>
<tr>
<td></td>
<td>age</td>
<td>-0.127</td>
<td>0.021</td>
<td>0.032</td>
<td>0.014</td>
</tr>
<tr>
<td></td>
<td>age$^2$ (10^{-3})</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>IB$_{PS2}$</td>
<td>1.127</td>
<td>0.138</td>
<td>-0.166</td>
<td>0.037</td>
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<td>IB$_{XBOX}$</td>
<td>1.673</td>
<td>0.204</td>
<td>0.091</td>
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<td></td>
<td>IB$_{GC}$</td>
<td>1.438</td>
<td>0.210</td>
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<tr>
<td>Software Parameters</td>
<td>age</td>
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<td>0.001</td>
<td>-0.197</td>
<td>0.003</td>
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<td>age$^2$ (10^{-3})</td>
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<td>0.000</td>
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<td>0.010</td>
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<td>0.002</td>
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<td></td>
<td>IB$_{XBOX}$</td>
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<tr>
<td></td>
<td>IB$_{GC}$</td>
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<td>0.045</td>
<td>0.003</td>
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<td>Log Likelihood</td>
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<td>-63486</td>
<td>-27654</td>
<td>-27649</td>
<td>-27649</td>
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<tr>
<td># HW Obs. ($n_{hw}$)</td>
<td>154</td>
<td>154</td>
<td>154</td>
<td>154</td>
<td>154</td>
</tr>
<tr>
<td># SW Obs. ($n_{sw}$)</td>
<td>57901</td>
<td>57901</td>
<td>57901</td>
<td>57901</td>
<td>57901</td>
</tr>
</tbody>
</table>

Notes: $\rho_{hw}$ and $\rho_{sw}$ are the estimated coefficients on the autoregressive processes for $\xi_{j,t}$ and $\eta_{j,k,t}$ in (15); $\alpha_0^{p,sw}$ is the software price sensitivity for the mean consumer in (9), and $\sigma_{\alpha_{p,sw}}$ denotes the degree of income heterogeneity (as discussed in section 3.4, $\alpha_0^{p,sw} = \sigma_\epsilon^{p,sw} = \sigma_{\alpha_{p,sw}} y_i$, where $y_i$ is consumer $i$’s annual household income in $’000s); $\sigma_\gamma$ is the standard deviation of consumer heterogeneity for gaming intensity $\alpha_\gamma$; $\beta_\gamma$ is the additional decay coefficient for future software utility given by (12); $D$ is the complementarity factor from owning multiple platforms in (13); and “Implied $\alpha_0^{p,hw}$” is the value hardware price sensitivity given by the restriction $\alpha_0^{p,hw} = \sigma_\epsilon^{p,sw} \alpha_0^{p,sw}$. For the coefficients $\alpha^2$ and $\alpha_w$ on hardware and software characteristics, $d_j$ are fixed effects for platform $j$, age and age$^2$ are monthly decay effects, and IB$_j$ are platform specific log installed base coefficients.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameter Estimate</th>
<th>Standard Error</th>
<th>Variable</th>
<th>Parameter Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hardware</td>
<td></td>
<td></td>
<td>Software</td>
<td></td>
<td></td>
</tr>
<tr>
<td>dFeb</td>
<td>0.191</td>
<td>0.065</td>
<td>dFeb</td>
<td>0.090</td>
<td>0.012</td>
</tr>
<tr>
<td>dMar</td>
<td>0.078</td>
<td>0.084</td>
<td>dMar</td>
<td>0.101</td>
<td>0.015</td>
</tr>
<tr>
<td>dApr</td>
<td>-0.266</td>
<td>0.095</td>
<td>dApr</td>
<td>-0.262</td>
<td>0.018</td>
</tr>
<tr>
<td>dMay</td>
<td>-0.533</td>
<td>0.100</td>
<td>dMay</td>
<td>-0.334</td>
<td>0.019</td>
</tr>
<tr>
<td>dJun</td>
<td>-0.085</td>
<td>0.104</td>
<td>dJun</td>
<td>0.087</td>
<td>0.020</td>
</tr>
<tr>
<td>dJuly</td>
<td>-0.413</td>
<td>0.105</td>
<td>dJuly</td>
<td>-0.011</td>
<td>0.020</td>
</tr>
<tr>
<td>dAug</td>
<td>-0.513</td>
<td>0.104</td>
<td>dAug</td>
<td>-0.017</td>
<td>0.020</td>
</tr>
<tr>
<td>dSep</td>
<td>-0.336</td>
<td>0.102</td>
<td>dSep</td>
<td>0.028</td>
<td>0.019</td>
</tr>
<tr>
<td>dOct</td>
<td>-0.554</td>
<td>0.097</td>
<td>dOct</td>
<td>-0.167</td>
<td>0.018</td>
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<tr>
<td>dNov</td>
<td>0.031</td>
<td>0.087</td>
<td>dNov</td>
<td>-0.045</td>
<td>0.016</td>
</tr>
<tr>
<td>dDec</td>
<td>1.037</td>
<td>0.066</td>
<td>dDec</td>
<td>1.011</td>
<td>0.012</td>
</tr>
<tr>
<td>All Systems</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Playstat</td>
<td></td>
<td></td>
<td>Gamecube</td>
<td></td>
<td></td>
</tr>
<tr>
<td>dFeb</td>
<td>0.086</td>
<td>0.008</td>
<td>dFeb</td>
<td>0.056</td>
<td>0.014</td>
</tr>
<tr>
<td>dMar</td>
<td>0.102</td>
<td>0.011</td>
<td>dMar</td>
<td>0.058</td>
<td>0.018</td>
</tr>
<tr>
<td>dApr</td>
<td>-0.185</td>
<td>0.013</td>
<td>dApr</td>
<td>-0.275</td>
<td>0.020</td>
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<tr>
<td>dMay</td>
<td>-0.216</td>
<td>0.014</td>
<td>dMay</td>
<td>-0.345</td>
<td>0.022</td>
</tr>
<tr>
<td>dJun</td>
<td>0.162</td>
<td>0.014</td>
<td>dJun</td>
<td>0.088</td>
<td>0.023</td>
</tr>
<tr>
<td>dJuly</td>
<td>0.087</td>
<td>0.014</td>
<td>dJuly</td>
<td>0.018</td>
<td>0.023</td>
</tr>
<tr>
<td>dAug</td>
<td>0.040</td>
<td>0.014</td>
<td>dAug</td>
<td>0.049</td>
<td>0.023</td>
</tr>
<tr>
<td>dSep</td>
<td>0.116</td>
<td>0.014</td>
<td>dSep</td>
<td>0.048</td>
<td>0.022</td>
</tr>
<tr>
<td>dOct</td>
<td>-0.095</td>
<td>0.013</td>
<td>dOct</td>
<td>-0.077</td>
<td>0.021</td>
</tr>
<tr>
<td>dNov</td>
<td>0.110</td>
<td>0.011</td>
<td>dNov</td>
<td>0.252</td>
<td>0.018</td>
</tr>
<tr>
<td>dDec</td>
<td>1.120</td>
<td>0.008</td>
<td>dDec</td>
<td>1.207</td>
<td>0.014</td>
</tr>
</tbody>
</table>

Notes: Coefficients on month dummies for both hardware and software from the full dynamic demand model (specification (v) in Table 3).
### Table 5: Regression of Software Title Fixed Effects

<table>
<thead>
<tr>
<th>Variable</th>
<th>All Titles</th>
<th>PS2 Only</th>
<th>XBOX Only</th>
<th>GC Only</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Parameter</td>
<td>Standard</td>
<td>Parameter</td>
<td>Standard</td>
</tr>
<tr>
<td>Exclusive, 1st Party</td>
<td>0.626</td>
<td>0.132</td>
<td>0.497</td>
<td>0.183</td>
</tr>
<tr>
<td>Exclusive, 3rd Party</td>
<td>-0.334</td>
<td>0.078</td>
<td>-0.356</td>
<td>0.100</td>
</tr>
<tr>
<td>PS2</td>
<td>-4.574</td>
<td>0.101</td>
<td>-4.536</td>
<td>0.126</td>
</tr>
<tr>
<td>Xbox</td>
<td>-3.962</td>
<td>0.107</td>
<td>-4.112</td>
<td>0.161</td>
</tr>
<tr>
<td>GC</td>
<td>-3.794</td>
<td>0.121</td>
<td>-3.645</td>
<td>0.179</td>
</tr>
<tr>
<td>Feb</td>
<td>-0.300</td>
<td>0.165</td>
<td>-0.177</td>
<td>0.256</td>
</tr>
<tr>
<td>Mar</td>
<td>-0.113</td>
<td>0.173</td>
<td>-0.177</td>
<td>0.215</td>
</tr>
<tr>
<td>Apr</td>
<td>-0.293</td>
<td>0.217</td>
<td>-0.379</td>
<td>0.293</td>
</tr>
<tr>
<td>May</td>
<td>-0.068</td>
<td>0.200</td>
<td>0.151</td>
<td>0.304</td>
</tr>
<tr>
<td>June</td>
<td>-0.497</td>
<td>0.193</td>
<td>-0.449</td>
<td>0.245</td>
</tr>
<tr>
<td>July</td>
<td>-0.225</td>
<td>0.206</td>
<td>-0.740</td>
<td>0.313</td>
</tr>
<tr>
<td>Aug</td>
<td>0.315</td>
<td>0.151</td>
<td>0.035</td>
<td>0.261</td>
</tr>
<tr>
<td>Sept</td>
<td>0.265</td>
<td>0.134</td>
<td>0.241</td>
<td>0.189</td>
</tr>
<tr>
<td>Oct</td>
<td>0.532</td>
<td>0.127</td>
<td>0.558</td>
<td>0.184</td>
</tr>
<tr>
<td>Nov</td>
<td>-0.032</td>
<td>0.147</td>
<td>0.021</td>
<td>0.187</td>
</tr>
<tr>
<td>Dec</td>
<td>0.137</td>
<td>0.220</td>
<td>0.216</td>
<td>0.284</td>
</tr>
</tbody>
</table>

### Notes:
OLS Regression of recovered software fixed effects $\alpha_w$ for each software title on dummy variables indicating whether or not it was exclusive, the platform it was released on, and the month of release. “Exclusive, 1st Party” indicates title was published by a platform provider; “Exclusive, 3rd Party” indicates the title was published by a third-party publisher.
Table 6: Estimated Hardware Own and Cross-Price Semi-Elasticities

<table>
<thead>
<tr>
<th>Model</th>
<th>PS2</th>
<th>XBOX</th>
<th>GC</th>
<th>Outside</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Static</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.157</td>
<td>-0.002</td>
<td>-0.002</td>
<td>-0.279</td>
</tr>
<tr>
<td>No Multihoming</td>
<td>XBOX</td>
<td>0.157</td>
<td>-0.001</td>
<td>-0.151</td>
</tr>
<tr>
<td>No Heterogeneity</td>
<td>GC</td>
<td>-0.001</td>
<td>0.158</td>
<td>-0.108</td>
</tr>
</tbody>
</table>

| (iii) |      |      |      |         |
| Dynamic|      |      |      |         |
|       | 0.166| -0.033| -0.029| -0.067 |
| No Multihoming| XBOX| 0.187| -0.011| -0.032 |
| No Heterogeneity| GC| -0.009| 0.192| -0.023 |

| (v)   |      |      |      |         |
| Dynamic|      |      |      |         |
|       | 0.199| -0.049| -0.038| -0.066 |
| No Multihoming| XBOX| 0.234| -0.016| -0.029 |
| Heterogeneity| GC| -0.013| 0.245| -0.020 |

Notes: Cell entries $i, j$, where $i$ indexes row and $j$ indexes column, provides the percentage change in market share with a permanent 10% decrease in the price of console $i$. 95% confidence intervals are provided in parenthesis below estimates.

Table 7: Estimated Software Own-Price Semi-Elasticities

<table>
<thead>
<tr>
<th>Model</th>
<th>DYN</th>
<th>MH</th>
<th>HET</th>
<th>PS2</th>
<th>XBOX</th>
<th>GC</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>0.181</td>
<td>0.752</td>
<td>0.511</td>
</tr>
</tbody>
</table>

| (ii)  | Yes | No | No  | 0.120| 0.409| 0.278|

| (iii) | Yes | Yes| No  | 0.097| 0.147| 0.086|

| (iv)  | Yes | No | Yes | 0.106| 0.414| 0.263|

| (v)   | Yes | Yes| Yes | 0.110| 0.165| 0.089|

Notes: Percentage change in total quantity sold of a top selling title on each console conditional on a permanent 10% decrease in the price of that title. The software titles are *Grand Theft Auto III* for the PS2, *Halo* for the Xbox, and *Super Smash Bros.* for the GC. 95% confidence intervals are provided in parenthesis below estimates.
Table 8: Hardware Elasticities from Losing A Top Title

<table>
<thead>
<tr>
<th>Model (i)</th>
<th>PS2</th>
<th>XBOX</th>
<th>GC</th>
<th>Outside</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static</td>
<td>PS2</td>
<td>-0.018</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>XBOX</td>
<td>0.001</td>
<td>-0.056</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>GC</td>
<td>0.000</td>
<td>0.000</td>
<td>-0.043</td>
</tr>
<tr>
<td></td>
<td>Outside</td>
<td>0.000</td>
<td>0.000</td>
<td>(-.038, -.043)</td>
</tr>
<tr>
<td>No Multihoming</td>
<td>XBOX</td>
<td>(.000, .000)</td>
<td>(-.066, -.050)</td>
<td>(.000, .000)</td>
</tr>
<tr>
<td>No Heterogeneity</td>
<td>GC</td>
<td>(.000, .000)</td>
<td>(.000, .000)</td>
<td>(-.038, -.043)</td>
</tr>
<tr>
<td>Dynamic</td>
<td>PS2</td>
<td>-0.024</td>
<td>0.007</td>
<td>0.006</td>
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<tr>
<td></td>
<td>XBOX</td>
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<td>-0.090</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>GC</td>
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<td>0.004</td>
<td>-0.064</td>
</tr>
<tr>
<td></td>
<td>Outside</td>
<td>0.003</td>
<td>0.004</td>
<td>(-.075, -.053)</td>
</tr>
<tr>
<td>No Multihoming</td>
<td>XBOX</td>
<td>(.006, .009)</td>
<td>(-.104, -.076)</td>
<td>(.007, .010)</td>
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<tr>
<td>No Heterogeneity</td>
<td>GC</td>
<td>(.003, .004)</td>
<td>(.003, .005)</td>
<td>(-.075, -.053)</td>
</tr>
<tr>
<td>Model (iii)</td>
<td>PS2</td>
<td>-0.022</td>
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<td>0.013</td>
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<td>XBOX</td>
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<tr>
<td></td>
<td>GC</td>
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<td>0.006</td>
<td>-0.067</td>
</tr>
<tr>
<td></td>
<td>Outside</td>
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<td>0.004</td>
<td>(-.085, -.054)</td>
</tr>
<tr>
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<td>XBOX</td>
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<td>(-.116, -.072)</td>
<td>(.014, .028)</td>
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<td>GC</td>
<td>(.003, .005)</td>
<td>(.004, .007)</td>
<td>(-.085, -.054)</td>
</tr>
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</table>

Notes: Cell entries $i, j$, where $i$ indexes row and $j$ indexes column, provides the percentage change in market share of console $j$ upon console $i$ losing a top software title. The software titles are Grand Theft Auto III for the PS2, Halo for the Xbox, and Super Smash Bros. for the GC. 95% confidence intervals are provided in parenthesis below estimates.

Table 9: Hardware Elasticities from Forced Compatibility of Software Titles

<table>
<thead>
<tr>
<th>DYN</th>
<th>MH</th>
<th>HET</th>
<th>PS2</th>
<th>XBOX</th>
<th>GC</th>
<th>Outside</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model (i)</td>
<td>No</td>
<td>No</td>
<td>No</td>
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<td>1.039</td>
<td>1.292</td>
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<tr>
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<td>(.150, .224)</td>
<td>-</td>
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<td></td>
</tr>
<tr>
<td>Model (ii)</td>
<td>Yes</td>
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<td>No</td>
<td>1.081</td>
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<td>(-.125, .916)</td>
<td>(-.966, -.854)</td>
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<tr>
<td>Model (iii)</td>
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<td>Yes</td>
<td>No</td>
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<td>0.413</td>
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<td></td>
<td>(1.823, 2.561)</td>
<td>(-.843, -.503)</td>
<td>(-.834, .894)</td>
<td>(-.972, -.940)</td>
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<tr>
<td>Model (iv)</td>
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<td>Yes</td>
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<td>0.044</td>
</tr>
<tr>
<td></td>
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<td>(.000, .106)</td>
<td>(-.380, -.257)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model (v)</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>0.745</td>
<td>-0.124</td>
<td>0.168</td>
</tr>
<tr>
<td></td>
<td>(.574, 1.10)</td>
<td>(-.195, -.082)</td>
<td>(.018, .270)</td>
<td>(-.543, -.329)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Percentage change in market share of each console subject to every software title “multihoming” and joining all three consoles. 95% confidence intervals are provided in parenthesis below estimates.
| Genre | Estimate | 95% CI | | Genre | Estimate | 95% CI | | Genre | Estimate | 95% CI |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (i) Action | PS2 | 0.000 | 0.000 | 0.000 | XBOX | -1.048 | -1.586 | 0.944 | GC | -1.159 | -2.156 | -0.220 |
| | PS2 & XBOX | 0.792 | 0.245 | 1.380 | | PS2 & GC | 0.083 | 0.049 | 0.129 | | XBOX & GC | -0.736 | -1.332 | 0.862 |
| | All 3 | 0.849 | 0.422 | 1.221 | | c_{PS2} | -0.247 | -0.332 | -0.009 | | GC | 2.127 | 0.535 | 4.537 |
| | # Titles | 241 | | | | # Titles | 100 | | | | # Titles | 103 |
| (ii) Family | PS2 | 0.000 | 0.000 | 0.000 | XBOX | 0.218 | 0.291 | 3.613 | GC | -0.131 | 0.009 | 0.038 |
| | PS2 & XBOX | -0.178 | -0.462 | 0.588 | | PS2 & GC | 1.724 | 0.341 | 1.724 | | XBOX & GC | 2.567 | 1.467 | 22.392 |
| | All 3 | 1.756 | 0.123 | 2.366 | | c_{PS2} | 0.255 | -0.007 | 0.761 | | GC | -0.228 | -0.321 | -0.004 |
| | # Titles | 103 | | | | # Titles | 197 | | | | # Titles | 163 |
| (iii) Fighting | PS2 | 0.000 | 0.000 | 0.000 | XBOX | -2.254 | -5.266 | 1.092 | GC | -0.131 | 0.009 | 0.038 |
| | PS2 & XBOX | 4.761 | -0.010 | 18.284 | | PS2 & GC | -0.228 | -0.321 | -0.004 | | XBOX & GC | 2.567 | 1.467 | 22.392 |
| | All 3 | 1.756 | 0.123 | 2.366 | | c_{PS2} | 0.255 | -0.007 | 0.761 | | GC | -0.228 | -0.321 | -0.004 |
| | # Titles | 103 | | | | # Titles | 197 | | | | # Titles | 163 |
| (iv) Platformer | PS2 | 0.000 | 0.000 | 0.000 | XBOX | 0.413 | 0.413 | 0.772 | GC | 0.470 | 0.470 | 1.899 |
| | PS2 & XBOX | -0.178 | -0.462 | 0.588 | | PS2 & GC | 1.724 | 0.341 | 1.724 | | XBOX & GC | 2.567 | 1.467 | 22.392 |
| | All 3 | 1.756 | 0.123 | 2.366 | | c_{PS2} | 0.255 | -0.007 | 0.761 | | GC | -0.228 | -0.321 | -0.004 |
| | # Titles | 103 | | | | # Titles | 197 | | | | # Titles | 163 |
| (v) Racing | PS2 | 0.000 | 0.000 | 0.000 | XBOX | 0.413 | 0.413 | 0.772 | GC | 0.470 | 0.470 | 1.899 |
| | PS2 & XBOX | -0.178 | -0.462 | 0.588 | | PS2 & GC | 1.724 | 0.341 | 1.724 | | XBOX & GC | 2.567 | 1.467 | 22.392 |
| | All 3 | 1.756 | 0.123 | 2.366 | | c_{PS2} | 0.255 | -0.007 | 0.761 | | GC | -0.228 | -0.321 | -0.004 |
| | # Titles | 103 | | | | # Titles | 197 | | | | # Titles | 163 |
| (vi) RPG | PS2 | 0.000 | 0.000 | 0.000 | XBOX | 0.413 | 0.413 | 0.772 | GC | 0.470 | 0.470 | 1.899 |
| | PS2 & XBOX | -0.178 | -0.462 | 0.588 | | PS2 & GC | 1.724 | 0.341 | 1.724 | | XBOX & GC | 2.567 | 1.467 | 22.392 |
| | All 3 | 1.756 | 0.123 | 2.366 | | c_{PS2} | 0.255 | -0.007 | 0.761 | | GC | -0.228 | -0.321 | -0.004 |
| | # Titles | 103 | | | | # Titles | 197 | | | | # Titles | 163 |
| (vii) Shooter | PS2 | 0.000 | 0.000 | 0.000 | XBOX | 0.413 | 0.413 | 0.772 | GC | 0.470 | 0.470 | 1.899 |
| | PS2 & XBOX | -0.178 | -0.462 | 0.588 | | PS2 & GC | 1.724 | 0.341 | 1.724 | | XBOX & GC | 2.567 | 1.467 | 22.392 |
| | All 3 | 1.756 | 0.123 | 2.366 | | c_{PS2} | 0.255 | -0.007 | 0.761 | | GC | -0.228 | -0.321 | -0.004 |
| | # Titles | 103 | | | | # Titles | 197 | | | | # Titles | 163 |
| (viii) Sports | PS2 | 0.000 | 0.000 | 0.000 | XBOX | 0.413 | 0.413 | 0.772 | GC | 0.470 | 0.470 | 1.899 |
| | PS2 & XBOX | -0.178 | -0.462 | 0.588 | | PS2 & GC | 1.724 | 0.341 | 1.724 | | XBOX & GC | 2.567 | 1.467 | 22.392 |
| | All 3 | 1.756 | 0.123 | 2.366 | | c_{PS2} | 0.255 | -0.007 | 0.761 | | GC | -0.228 | -0.321 | -0.004 |
| | # Titles | 103 | | | | # Titles | 197 | | | | # Titles | 163 |
| (ix) Other | PS2 | 0.000 | 0.000 | 0.000 | XBOX | 0.413 | 0.413 | 0.772 | GC | 0.470 | 0.470 | 1.899 |
| | PS2 & XBOX | -0.178 | -0.462 | 0.588 | | PS2 & GC | 1.724 | 0.341 | 1.724 | | XBOX & GC | 2.567 | 1.467 | 22.392 |
| | All 3 | 1.756 | 0.123 | 2.366 | | c_{PS2} | 0.255 | -0.007 | 0.761 | | GC | -0.228 | -0.321 | -0.004 |
| | # Titles | 103 | | | | # Titles | 197 | | | | # Titles | 163 |

Notes: Estimates of $\theta_C$ used to specify porting costs in (22) (units in $\text{M}$), separately estimated by genre. Instruments: constant, title fixed effect $\alpha^w_k$, inverse of expected quantity sold (M). 95% confidence intervals are constructed taking 80 sample draws from the empirical distribution of the moment inequalities and re-estimating costs. Values of title fixed effects $\{\alpha^w_k\}$ are scaled to be within $[1, 10]$. $a$ No titles observed to have chosen this strategy, so no upper bound is identified. During computation of counterfactual regimes, costs are assumed to be arbitrarily high for this particular genre and strategy choice.
Table 11: Dynamic Network Formation Game: Predicted Fit of Model

<table>
<thead>
<tr>
<th></th>
<th>Observed Data</th>
<th>Predicted Data</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Estimate</td>
<td>Conf. Interval</td>
</tr>
<tr>
<td><strong>Installed Base (M)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PS2</td>
<td>30.07</td>
<td>29.40</td>
<td>27.91</td>
<td>29.59</td>
</tr>
<tr>
<td>XB</td>
<td>13.32</td>
<td>12.86</td>
<td>12.48</td>
<td>13.32</td>
</tr>
<tr>
<td>GC</td>
<td>9.83</td>
<td>11.02</td>
<td>10.53</td>
<td>11.22</td>
</tr>
<tr>
<td><strong>% Market Shares</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PS2</td>
<td>56.50</td>
<td>55.18</td>
<td>53.92</td>
<td>55.54</td>
</tr>
<tr>
<td>XB</td>
<td>25.03</td>
<td>24.13</td>
<td>23.57</td>
<td>25.67</td>
</tr>
<tr>
<td>GC</td>
<td>18.47</td>
<td>20.69</td>
<td>20.15</td>
<td>21.46</td>
</tr>
<tr>
<td><strong># of Titles</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PS2</td>
<td>1161</td>
<td>953</td>
<td>559</td>
<td>1031</td>
</tr>
<tr>
<td>XB</td>
<td>749</td>
<td>943</td>
<td>787</td>
<td>1067</td>
</tr>
<tr>
<td>GC</td>
<td>487</td>
<td>795</td>
<td>556</td>
<td>884</td>
</tr>
<tr>
<td><strong># of “Hit” Titles</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Sales &gt; 100K)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PS2</td>
<td>578</td>
<td>585</td>
<td>427</td>
<td>613</td>
</tr>
<tr>
<td>XB</td>
<td>296</td>
<td>293</td>
<td>272</td>
<td>309</td>
</tr>
<tr>
<td>GC</td>
<td>290</td>
<td>245</td>
<td>220</td>
<td>251</td>
</tr>
<tr>
<td><strong># of “Hit” Titles</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Sales &gt; 1M)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PS2</td>
<td>67</td>
<td>62</td>
<td>61</td>
<td>64</td>
</tr>
<tr>
<td>XB</td>
<td>9</td>
<td>8</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>GC</td>
<td>8</td>
<td>11</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td><strong>Total Titles Sold (M)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PS2</td>
<td>305.09</td>
<td>297.22</td>
<td>252.78</td>
<td>304.15</td>
</tr>
<tr>
<td>XB</td>
<td>118.05</td>
<td>115.07</td>
<td>105.26</td>
<td>123.93</td>
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<td>GC</td>
<td>79.17</td>
<td>103.45</td>
<td>87.66</td>
<td>107.44</td>
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</table>

Notes: Predicted data obtained by fixing strategic platform choices for first-party titles, but allowing third-party titles to re-optimize. Porting cost estimates are from table 10. Confidence intervals are computed by redrawing from the estimated porting cost distribution for multiple sets of instruments, and recomputing a new equilibrium.
Table 12: Counterfactual: Banning Vertical Integration and Exclusivity

<table>
<thead>
<tr>
<th>Observed Data</th>
<th>(i) CF #1: First Party Titles</th>
<th>(ii) CF #2: No FP Titles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>Conf. Interval</td>
</tr>
<tr>
<td>Installed Base (M)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PS2</td>
<td>30.07</td>
<td>58.02</td>
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<td>XB</td>
<td>13.32</td>
<td>8.70</td>
</tr>
<tr>
<td>GC</td>
<td>9.83</td>
<td>10.06</td>
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<tr>
<td>% Market Shares</td>
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<td></td>
</tr>
<tr>
<td>PS2</td>
<td>56.50</td>
<td>75.56</td>
</tr>
<tr>
<td>XB</td>
<td>25.03</td>
<td>11.32</td>
</tr>
<tr>
<td>GC</td>
<td>18.47</td>
<td>13.10</td>
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<tr>
<td>Number of Titles</td>
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<td></td>
</tr>
<tr>
<td>PS2</td>
<td>1161</td>
<td>1175</td>
</tr>
<tr>
<td>XB</td>
<td>749</td>
<td>1169</td>
</tr>
<tr>
<td>GC</td>
<td>487</td>
<td>936</td>
</tr>
<tr>
<td># of “Hit” Titles (Sales &gt; 100K)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PS2</td>
<td>578</td>
<td>997</td>
</tr>
<tr>
<td>XB</td>
<td>296</td>
<td>187</td>
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<tr>
<td>GC</td>
<td>290</td>
<td>376</td>
</tr>
<tr>
<td># of “Hit” Titles (Sales &gt; 1M)</td>
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<td></td>
</tr>
<tr>
<td>PS2</td>
<td>67</td>
<td>325</td>
</tr>
<tr>
<td>XB</td>
<td>9</td>
<td>5</td>
</tr>
<tr>
<td>GC</td>
<td>8</td>
<td>32</td>
</tr>
<tr>
<td>Total Titles Sold (M)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PS2</td>
<td>305.09</td>
<td>1474.79</td>
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<tr>
<td>XB</td>
<td>118.05</td>
<td>69.19</td>
</tr>
<tr>
<td>GC</td>
<td>79.17</td>
<td>178.32</td>
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</table>

Notes: Counterfactual results allow all titles to re-optimize and choose the optimal set of platforms. CF #1 assumes first-party titles are still produced and enter the market; CF #2 assumes all first-party titles are eliminated. Estimates are computed using porting cost estimates from table 10. Confidence intervals are computed by redrawing from the estimated porting cost distribution for multiple sets of instruments and recomputing a new equilibrium.
Table 13: Estimated Parameters of Negative Binomial Regression

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameter</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln((\mu))</td>
<td>Constant</td>
<td>1.423</td>
</tr>
<tr>
<td></td>
<td>(Q_{jt})</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>(Age)</td>
<td>-0.025</td>
</tr>
<tr>
<td></td>
<td>(Age^2(10^{-2}))</td>
<td>0.032</td>
</tr>
<tr>
<td></td>
<td>(d_{Feb})</td>
<td>0.476</td>
</tr>
<tr>
<td></td>
<td>(d_{Mar})</td>
<td>1.109</td>
</tr>
<tr>
<td></td>
<td>(d_{Apr})</td>
<td>0.416</td>
</tr>
<tr>
<td></td>
<td>(d_{May})</td>
<td>0.424</td>
</tr>
<tr>
<td></td>
<td>(d_{Jun})</td>
<td>0.789</td>
</tr>
<tr>
<td></td>
<td>(d_{Jul})</td>
<td>0.322</td>
</tr>
<tr>
<td></td>
<td>(d_{Aug})</td>
<td>0.800</td>
</tr>
<tr>
<td></td>
<td>(d_{Sep})</td>
<td>1.597</td>
</tr>
<tr>
<td></td>
<td>(d_{Oct})</td>
<td>1.492</td>
</tr>
<tr>
<td></td>
<td>(d_{Nov})</td>
<td>1.700</td>
</tr>
<tr>
<td></td>
<td>(d_{Dec})</td>
<td>0.743</td>
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<tr>
<td>ln((\alpha))</td>
<td>Constant</td>
<td>-0.245</td>
</tr>
<tr>
<td></td>
<td>(d_{PS2})</td>
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</tr>
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<td></td>
<td>(d_{XBOX})</td>
<td>-2.502</td>
</tr>
<tr>
<td></td>
<td>(d_{GC})</td>
<td>-3.000</td>
</tr>
</tbody>
</table>

Num. Obs. 458

Notes: See Appendix A. Coefficients are from negative binomial regression of number of titles released at time \(t\) for seven consoles, including four fifth-generation consoles. Only coefficients for sixth-generation consoles are reported for ln(\(\alpha\)).

Table 14: Software Price Regressions

<table>
<thead>
<tr>
<th>Explanatory Variable</th>
<th>Parameter</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>price_{t-1}</td>
<td>0.922</td>
<td></td>
</tr>
<tr>
<td>(price_{t-1})^2</td>
<td>0.659</td>
<td></td>
</tr>
<tr>
<td>Qt-1 (10^{-3})</td>
<td>0.560</td>
<td></td>
</tr>
<tr>
<td>(d_{Feb})</td>
<td>-0.108</td>
<td></td>
</tr>
<tr>
<td>(d_{Mar})</td>
<td>0.569</td>
<td></td>
</tr>
<tr>
<td>(d_{Apr})</td>
<td>0.497</td>
<td></td>
</tr>
<tr>
<td>(d_{May})</td>
<td>-0.315</td>
<td></td>
</tr>
<tr>
<td>(d_{Jun})</td>
<td>1.227</td>
<td></td>
</tr>
<tr>
<td>(d_{Jul})</td>
<td>0.249</td>
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<tr>
<td>(d_{Aug})</td>
<td>-0.296</td>
<td></td>
</tr>
<tr>
<td>(d_{Sep})</td>
<td>-0.062</td>
<td></td>
</tr>
<tr>
<td>(d_{Oct})</td>
<td>1.299</td>
<td></td>
</tr>
<tr>
<td>(d_{Nov})</td>
<td>0.671</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.671</td>
<td></td>
</tr>
</tbody>
</table>

R^2 0.924

# Observations 58337

Notes: See Appendix B. OLS Regression of prices on lagged prices and quantities for each software title, pooled across all platforms.
Figure 1: Hardware Quantities, Installed Bases, and Prices

Notes: In the top graph, bars represent the total number hardware consoles sold across all three platforms in each month in thousands (scale on left); lines graphs indicate the total installed base for each console in millions (scale on right). The bottom graph provides average monthly (nominal) prices faced by consumers in retail stores for each platform.
Figure 2: Estimation Algorithm

Notes: Computation algorithm described in subsection 3.3 to compute log-likelihood function $\mathcal{L}(\theta)$ given by (16). “BLP Contraction Mapping” refers to the contraction mapping given by (30) introduced in Berry, Levinsohn, and Pakes (1995).
Figure 3: Evolution of Installed Bases

Notes: Implied evolution of installed bases for each console for consumers with different values of $\alpha^\gamma$. Darkest area at the bottom corresponds to the highest quintile of the distribution; lightest area at top corresponds to lowest quintile.
Figure 4: Estimated Values of $\{\delta_{j,t,0}\}_{j,t}$

[Graph showing estimated values for PS2, Xbox, and GC for different time periods, with notes indicating realized values of hardware mean-utility $\delta$ for mean consumer at inventory state $i = 0$ (no previous purchase) implied by the full demand model.]

Notes: Realized values of hardware mean-utility $\delta$ for mean consumer at inventory state $i = 0$ (no previous purchase) implied by the full demand model.

Figure 5: Difference Between Estimated $\delta_{j,t+1,0}$ and Predicted Value $E[\delta_{j,t+1,0}|\{\delta_{j,t,0}\}_{j,t}, m(t)]$

[Graph showing the difference between estimated and predicted values for PS2, Xbox, and GC for different time periods, with notes indicating errors between realized and predicted values of hardware mean-utility $\delta$ for mean consumer with no inventory using the estimated Markov transition process given by (19).]

Notes: Errors between realized and predicted values of hardware mean-utility $\delta$ for mean consumer with no inventory using the estimated Markov transition process given by (19).
Figure 6: Fit of Model: \{\nu_{j,t}^{hw}\}_{j,t}

Notes: Predicted residuals in hardware unobserved characteristics from full demand model: $\nu_{j,t}^{hw} \equiv \xi_{j,t} - \rho^{hw} \xi_{j,t}$. 
Notes: See Appendix A. Solid lines indicate actual number of titles \( q_{j,t} \) released for each platform in a given month; dashed lines indicate the 5% and 95% percentiles of the negative binomial distribution with parameters given in table 13 as a function of total software titles available in the previous period \( Q_{j,t-q} \).
References


