

# Optimal Public Investment and Fiscal Policy\*

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Despite the important role public investments play in an economy, only a limited amount of theoretical work has been done on the behavior of the public sector as an investing agent. The present paper attempts to fill this gap by formulating a simple dynamic model and then applying the optimal taxation approach to it to investigate the optimal paths of public investment and other fiscal instruments. Assuming labor supply to be inelastic, this paper examines the steady-state value of public capital stock and its social rate of return and the optimal rate of income tax which is used to finance the investment project.

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## 1. Introduction

Despite the important role public investments play in an economy, only a limited amount of theoretical work has been done on the behavior of the

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public sector as an investing agent. For decades most public economists have recommended to their policy-makers a rule-of-thumb method, the cost-benefit approach, to determine the desirability of undertaking a public project. However, the application of this pragmatic approach has usually encountered great difficulty since it involves quantities that are not measurable or have no market price.

In this paper we shall formulate a simple dynamic model of public investment and then apply the optimal taxation approach to it to investigate the paths of this variable and other fiscal instruments. The logic of using this approach is as follows: In a decentralized economy where the private sector is free to pursue its own interest, the government, subject to the technological constraint and resource availability, will use all policy instruments at its disposal to maximize a social welfare function (however it is defined). This is exactly the problem the optimal tax approach can be appropriately used to address.

The present paper bears some similarity to the seminal work by Arrow and Kurz (1969; 1970; chs. 7 and 8), though they differ in at least two important respects. First, by assuming fixed private capital, the model specified here is able to reduce the dynamic problem into a two-dimensional setting, the public investment and its shadow price. This simplification is necessary for using the phase-diagram technique to trace the optimal paths of public investment and other variables at every instant of time. Second, we assume that the public investment can be financed by increasing revenues from taxes on income and consumption or decreasing public consumption. With these assumptions, which seem reasonable, the economy considered will be able to attain the first-best optimum rather than a second-best equilibrium.

## **2. The Model**

The model economy we consider consists of the two sectors, private and

public. All economic units in the private sector are identical; we can therefore focus on the behavior of a representative individual. He lives infinitely and has perfect foresight about the levels and variations of all relevant economic variables. Given the variables he treats as exogenous, the individual will choose a consumption-saving plan to maximize an intertemporal objective function, subject to an instantaneous income constraint. On the other hand, the public sector or government, taking as given the individual's conditions for optimality, will maximize a social welfare function with respect to the public investment and other fiscal policy instruments, subject, of course, to an instantaneous budget equation.

### **(1) The Private Sector**

Let the individual's utility function be  $U(C,K,G)$ , where  $C$  is consumption,  $K$  the services from the public capital stock, and  $G$  the consumption-type governmental expenditures.<sup>1</sup> For simplicity sake,  $U$  is assumed to be concave, homogeneous of degree one, and twice differentiable with respect to its arguments. The intertemporal objective function the individual attempts to maximize can thus be specified as

$$\int_0^{\infty} e^{-rs} U(C,K,G) ds,$$

where  $r$  is the net-of-income-tax rate of interest and  $s$  time variable. As is well known in cost-benefit analysis, the net rate of interest is always taken to be the private rate of discount while the gross rate is considered as the social rate of discount. Furthermore, Hodrick (1982) and Turnovsky (1987) have pointed out that when an individual's time preference rate is constant, the net rate of interest is the only value which is consistent with the ultimate attainment of a steady state equilibrium. Since the main focus of this paper is primarily on the long run situation,  $r$  will be used as the discount rate and

assumed to be constant.<sup>2</sup>

Output per capita is generated using  $K$  and labor which is assumed to be inelastic as is implicit in the production function  $F(K)$ .<sup>3</sup> The marginal productivity of capital is positive and diminishing with  $K$ . Assuming the general price to be fixed at unity, the income generated from sales of output is equal to  $F(K)$  as well. In addition, the individual also earns interest income from holding the government bond,  $iB$ , where  $i$  is the gross (before-tax) rate of interest and  $B$  the amount of the government bond. Both types of income are taxed at rate  $t$ , and the private consumption is also subject to an indirect tax at rate  $\tau$ .

The individual's instantaneous income constraint is then given by

$$(1 + \tau)C + \dot{B} = (1 - t) [F(K) + iB], \quad (2)$$

where  $B (\equiv dB/ds)$  denotes the change in bond holding. Equation (2) states that at any instant of time the individual's consumption plus saving equals his disposable income.<sup>4</sup>

In determining his optimal consumption-saving plan, the individual will take  $K$ ,  $G$ ,  $\tau$ ,  $t$ , and  $i$  as given and choose the value of  $C$  to maximize (1), subject to the constraint (2). This is a standard dynamic programming problem; we shall conduct the analysis in section 3.1.

## **(2) The Public Sector**

Since all individuals are identical, the social welfare can be considered to coincide with the representative individual's utility. The objective function the government will maximize is thus specified as

$$\int_0^{\infty} e^{-rs} U(C, K, G) ds, \quad (3)$$

where the social rate of discount is the gross rate of interest.

With regards to the constraint facing the public sector, it is assumed that

the public capital is provided freely to the private sector<sup>5</sup> and that the budget deficit is financed completely by government bond. With  $I$  denoting gross public investment, the budget constraint will be

$$\dot{B} = G + I + iB - t [F(K) + iB] - \tau C, \quad (4)$$

where  $\dot{B} \geq 0$ . The equation above states that the amount of new bond issued (or old bond withdrawn) at any instant of time equals total public spendings minus tax revenues.

Assuming the public capital stock to depreciate at a constant rate,  $\delta$ , net investment is equal to gross investment minus depreciation, or

$$\dot{K} = I - \delta K. \quad (5)$$

Now the problem facing the public sector can be summarized as follows: Given the optimal value of  $C$  the individual chooses, the government will select  $G$ ,  $I$ ,  $t$ , and  $\tau$  to maximize (3), subject to (4) and (5).

### **(3) The Economy**

Substituting (4) into (2), we have

$$F(K) = C + I + G. \quad (6)$$

Equation (6) defines the constraint the economy faces; at any instant of time, aggregate supply equals aggregate demand, the sum of private consumption, gross investment, and other governmental expenditures. It follows that any saving or dissaving by the private sector must imply a budget deficit or surplus of equal amount by the government. Though intuitively obvious, this simple truth has an important bearing on the stability of the model used here.

## **3. The Short- run Equilibrium**

### **(1) Choice of Consumption and Saving**

Begin with the private sector. Treating  $C$  as the control variable and  $B$  the state variable, we can easily obtain from (1) and (2) the following first-order conditions:<sup>6</sup>

$$U_C(C,K,G) = p(1 + \tau), \quad (7a)$$

$$p = 0; \text{ i.e., } p = \text{constant}, \quad (7b)$$

where  $U_C = \partial U / \partial C$  and  $p$  is the underdetermined multiplier associated with Eq. (2). In addition, there is the transversality condition

$$\lim_{s \rightarrow \infty} e^{-rs} p B = 0. \quad (7c)$$

Since  $p$  can be interpreted as the shadow price of saving (measured in terms of utility), (7a) simply states that the optimal value of  $C$  equates the marginal benefit of consumption,  $U_C/p$ , with the marginal cost,  $(1 + \tau)$ . This equation can be solved for  $C$  to obtain

$$C = C(p; K, G, \tau). \quad (8)$$

With  $p$  constant, (8) implies that  $C$  is invariant with respect to time.

It can also be observed that  $C$  is independent of  $t$ . This is so because variations in  $t$  do not affect the marginal benefit of consumption or its marginal cost. It follows that an increase in the rate of the income tax only reduces private saving.

With  $C$  constant, the solution for the individual's bond holding, obtained by integrating (2), is

$$B = e^{rs} \left[ B_0 + \frac{f(K) - (1 + \tau)C}{r} (1 - e^{-rs}) \right], \quad (9)$$

where  $f = (1 - t)F$  and  $B_0$  is the initial stock of bond holding. For the transversality condition in (7c) to hold, we require

$$B_0r + f(K) = (1 + \tau)C. \quad (10)$$

That is, at any instant of time the individual's gross consumption equals his personal disposable income. From this we conclude that private saving must be zero.

As stated previously, zero private saving implies balanced budget in the public sector. The reason that the government must maintain its budget in balance is straightforward. The economy in question is basically a stagnant one. Any attempt by the public sector to run a deficit and finance it by borrowing will result in increasing interest payments as debt accumulates over time.

Summing up, (7a) and (10) specify the equilibrium of the private sector. The former equation describes the usual marginal condition for consumer optimality. The latter is the budget constraint; with the individual always being in equilibrium, no accumulation of asset occurs.

## **(2) Determination of Optimal Fiscal Instruments**

Now turn attention to the public sector. By substituting  $\dot{B} = 0$  and  $B = B_0$  from (10) to (4), and making use of (5), we have

$$\dot{K} = F(K) - \delta K - G - C, \quad (11)$$

which is the constraint facing the economy. The government will maximize (3), subject to (8), (10), and (11). The Hamiltonian function for this dynamic programming can be written as

$$H = e^{-rs} \{ U(C,K,G) + \lambda [B_0r + f(K) - (1 + \tau)C] + q [F(K) - \delta K - G - C] \}, \quad (12)$$

where  $\tau$  is the undetermined multiplier associated with (10) and  $q$  the shadow price of the public investment.

Treating  $G$ ,  $t$ , and  $\lambda$  as control variables and  $K$  as the state variable, we derive from (12) the following optimality conditions:

$$U_C \frac{\partial C}{\partial G} + U_G + \tau(1+\tau) \frac{\partial C}{\partial G} - q \left( \frac{\partial C}{\partial G} + 1 \right) = 0, \quad (13a)$$

$$\lambda(i + F') = 0, \quad (13b)$$

$$U_C \frac{\partial C}{\partial \tau} - q \frac{\partial C}{\partial \tau} - \tau \left[ C + (1+\tau) \frac{\partial C}{\partial \tau} \right] = 0, \quad (13c)$$

$$q = q \left( i + \delta - F' + \frac{\partial C}{\partial K} \right) - \tau \left[ (1-t)F' - (1+\tau) \frac{\partial C}{\partial K} \right] - (U_C \frac{\partial C}{\partial K} + U_K). \quad (13d)$$

In addition, the transversality condition is given by

$$\lim_{s \rightarrow \infty} e^{-rs} qK = 0. \quad (14)$$

Equations (13a)-(13c) and (10) specify the short-run equilibrium of the economy. At any instant of time,  $K$  and  $q$  are given; then these equations yield essentially the “derived demands” for  $G$ ,  $t$ ,  $\tau$ , and  $\lambda$  as functions of  $K$  and  $q$ . On the other hand, (13d) and (11) constitute a simultaneous system of the first-order differential equations. They are to determine the optimal paths of public capital stock,  $K$ , and the shadow price of investment,  $q$ . We shall first analyze the short-run equilibrium and then in the section below address the dynamic problem.

Beginning with (13b), we note that since  $i + F' > 0$ ,  $\lambda$  must equal zero. To understand the meaning of this condition, we assume that the government impose a lump-sum tax designated by  $T$ . Equation (10) is thus replaced by

$$B_0 i + F(K) = (1+\tau)C + T,$$



while all other relevant equations remain the same. The optimality condition for the lump-sum tax can be easily verified to be  $\lambda = 0$ . It follows that in the system summarized by (13a)-(13c) and (10), the income tax is in essence equivalent to a lump-sum tax. It should be pointed out that the neutrality of the income tax is conditioned on the inelasticity of the labor supply.<sup>7</sup>

With  $\tau = 0$ , (13a) and (13c) are simplified to

$$U_C = U_G = q. \tag{15}$$

That is, the optimal values of  $G$  and  $\tau$  should be such that the marginal utility of consumption equals the marginal utility of governmental consumption expenditures and both of them are equal to the shadow price of the public investment. Graphically, assuming  $U_{CG}$  to be zero, the value of  $C$  should exceed, equal, or fall short of the level of  $G$  if the  $U_C$  curve lies everywhere above, coincidentally with, or below the  $U_G$  curve.

Substitution of (7a) into (15) gives

$$\tau^* = \frac{q}{p} - 1,$$

where  $\tau^*$  is the optimal value of  $\tau$ . It follows that  $\tau^* \geq 0$  as  $q \geq p$ . In a steady state,  $q$  is constant and equal to  $p$ . This must be so because, with  $K = 0$ , the constant value of the net national product,  $F(K) - \delta K$ , as given in (11), is divided between private and governmental consumption,  $C$  and  $G$ . Efficiency requires the commodity to be allocated in such a way that the marginal valuation of private consumption,  $pU_C$ , be equal to that of public consumption,  $qU_G$ . Making use of (15), the condition above can be met if  $p = q$ . On the other hand, during the course to an equilibrium, the value of  $q$  changes continually as  $q = 0$  (see Figure 1). Therefore,  $q$  can be greater or smaller than  $p$  and, accordingly, the value of  $\tau^*$  can be positive or negative.

Solving (15) we have the optimal values of  $G$  and  $\tau$  as follows:

$$G = G(K, q), \tag{16a}$$

$$\tau = \tau(K, q), \tag{16b}$$

From the strict concavity of  $U$  and  $F$ , these maximizing values are unique.

Total differentiation of (15), after rearrangement and simplification, yields

$$\begin{pmatrix} -(U_{GG} + U_{CG} \frac{\partial C}{\partial G}) & -U_{CG} \frac{\partial C}{\partial \tau} \\ 0 & U_{CC} \frac{\partial C}{\partial \tau} \end{pmatrix} \begin{pmatrix} dG \\ d\tau \end{pmatrix} = \begin{pmatrix} -dq + (U_{KG} + U_{CG} \frac{\partial C}{\partial K})dK \\ -dq \end{pmatrix} \tag{17}$$

where the condition that  $U_{CC} (\partial C/\partial J) + U_{jC} = 0, j = G, K$ , is used.<sup>8</sup> By assumption,  $U$  is strictly concave and hence  $U_{CC} < 0$  and  $U_{CC}U_{GG} > U_{CG}^2$ . All the principal minors of the matrix on the left-hand side of (17) can readily be proven to be positive. This matrix is therefore a P matrix. By the Gale-Nikaido univalence theorem, (17) can be solved uniquely for all endogenous variables at any given set of values for the exogenous variables.<sup>9</sup> We can therefore conduct the comparative static analysis of the short-run equilibrium.

The following equations are obtained from (17):

$$\frac{\partial G}{\partial q} = \frac{U_{CC} - U_{CG}}{U_{CC}(U_{CG} + U_{CG} \frac{\partial C}{\partial G})}, \quad (18a)$$

$$\frac{\partial G}{K} = - \frac{U_{KG} + U_{CG} \frac{\partial C}{\partial K}}{U_{GG} + U_{CG} \frac{\partial C}{\partial G}}, \quad (18b)$$

$$\frac{\partial \tau}{\partial q} = \frac{1}{U_{CC} \frac{\partial C}{\partial \tau}} < 0, \quad (18c)$$

and  $\partial \tau / \partial K = 0$ .<sup>10</sup> While the impact on  $\tau$  of a change in  $q$  or  $K$  is certain, the effect on  $G$  is not without ambiguity. For analytical purpose we further impose two conditions on the utility function: (a)  $U_{CC} - U_{CG} < 0$ , and (b)  $U_{GK} + U_{CC}(\partial C / \partial K) = (U_{CC}U_{GK} - U_{GC}U_{CK}) / U_{CC} > 0$ .

The “dominant diagonal” assumption (a) would seem plausible. First, government’s expenditures (such as highways, defence, law enforcement, sanitation, etc.) would typically increase satisfaction from *private* consumption of automobile or homeownership. Thus,  $U_{CG} \geq 0$ . Even if  $U_{CG} < 0$ , we would hardly expect that in general the marginal utility of consumption to fall more with an additional unit of public goods than with an additional unit of private goods.

The assumption (b) that stipulates  $U_{GK} > 0$  can be justified on the ground that a public investment (for instance, highways) entails increased spending on personnel, facilities, and equipment for repair and maintenance. The assumption (b) or the “dominant direct effect” hypothesis is not unrealistic since, as may be expected, the direct impact on the marginal utility of governmental expenditures of a change in public capital,  $U_{GK}$ , would overwhelm the

indirect effect of that change on private consumption which in turn would have impact on governmental expenditures,  $U_{GC}(\partial C/\partial K)$ .

With assumptions (a) and (b), Eqs. (18a) and (18b) imply that  $\partial G/\partial q < 0$  and  $\partial G/\partial K > 0$ . That is, a rise in the shadow price of public investment or a fall in the public capital reduces the demand for government consumption expenditures.

#### 4. The Long- run Equilibrium

To study the dynamic nature of the optimal path of public investment, we return to the system of differential equations, (11) and (13d). With  $\tau = 0$ , (13d) is reduced to

$$q = q(i + \delta - F') - U_K. \quad (19)$$

##### (1) Analysis of the Optimal Path

Starting with the general shape of the  $K = 0$  curve, we can derive from (11) that

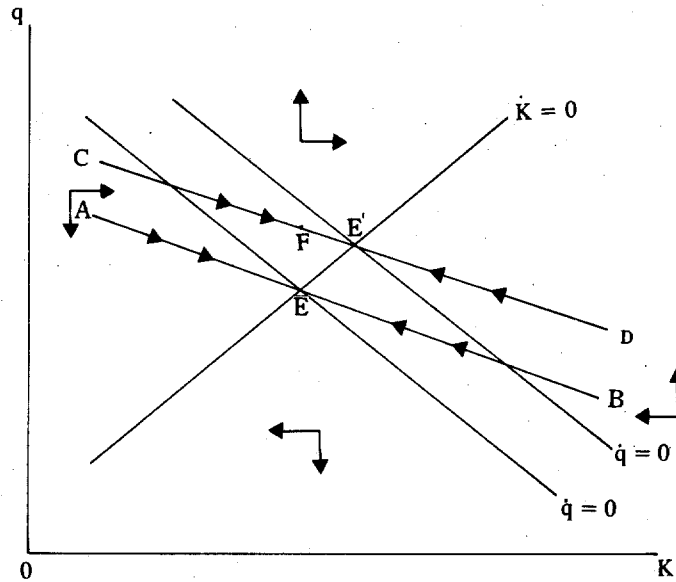
$$\left. \frac{\partial q}{\partial K} \right|_{K=0} = \frac{b_{12}}{b_{11}}, \quad (20a)$$

$$\text{where } b_{11} = \left( 1 + \frac{\partial C}{\partial G} \right) \frac{\partial G}{\partial q} + \frac{\partial C}{\partial \tau} \frac{\partial \tau}{\partial q},$$

$$b_{12} = \left[ \delta + \left( 1 + \frac{\partial C}{\partial G} \right) \frac{\partial G}{\partial K} + \frac{\partial C}{\partial K} \right].$$

In the equation above  $b_{11}$  is definitely negative but the sign of  $b_{12}$  is uncertain. Increasing  $K$  by one unit expands production by  $F'(K)$  and raises aggregate demand indicated by terms in the brackets on the right-hand side of the equality sign. If it is assumed that  $b_{12} \leq 0$ ,<sup>11</sup> then the  $K = 0$  locus slopes upward, as

depicted in Figure 1.



Intuitively, the  $K = 0$  curve is upward sloping because a rise in  $q$  decreases  $G$  and raises  $\tau$ , both of which tend to lower aggregate demand. To offset the fall in aggregate demand  $K$  must be expanded. With the assumption given above, an increase in  $K$  would raise aggregate demand by an amount greater than it would increase aggregate supply.

For points above the  $K = 0$  locus, output exceeds aggregate demand so that net investment is positive. For points below the  $K = 0$  locus, net investment is negative.

It can also be obtained from (19) that when  $\dot{q} = 0$ ,

$$\left. \frac{\partial q}{\partial K} \right|_{\dot{q}=0} = \frac{b_{22}}{b_{21}}, \quad (20b)$$

where  $b_{21} = i + \delta - F' - (U_{KC} \frac{\partial C}{\partial G} + U_{KG}) \frac{\partial G}{\partial q}$ ,

$$b_{22} = qF'' + U_{KK} + U_{KC} \frac{\partial C}{\partial K} + (U_{KG} + U_{KC} \frac{\partial C}{\partial G}) \frac{\partial G}{\partial K}.$$

Under assumption (b),  $b_{21}$  is unambiguously positive but the sign of  $b_{22}$  remains to be determined. It should be noted that  $U_{KK}$  represents the direct impact of changes in  $K$  on the marginal utility of  $K$ , while  $U_{KC}(\partial C/\partial K)$  and  $(U_{KG} + U_{KC}(\partial C/\partial G))\partial G/\partial K$  indicate the indirect effect of changes in  $K$ . By the assumption of the dominant direct effect, the sum of these terms is negative and hence  $b_{22} < 0$ . It follows that the  $q = 0$  locus is downward sloping as shown in Figure 1.

Intuitively, the slope of the  $q = 0$  curve is negative because an increase in  $K$  reduces the marginal benefit of  $K$  and has no effect on its marginal cost. To offset this fall in the marginal benefit calls for a decrease in  $q$  which tends to increase  $G$  and, in turn, raise the individual's utility. Furthermore, as can readily be seen from (19), for any point above the  $\dot{q} = 0$  locus,  $q$  is rising; for any point below the  $\dot{q} = 0$  locus,  $q$  is falling.

Figure 1 displays the  $\dot{K} = 0$  and  $\dot{q} = 0$  loci and vectors of motion corresponding to (11) and (19). The steady-state equilibrium is seen to be a saddle point. Given any level of the public capital stock  $K$ , there is a unique value of  $q$  which lies on the path to the steady state. The transversality condition (14) requires that the system not blow up asymptotically, i.e., that it lies on the stable path,  $CD$ , to the long-run equilibrium.

Finally, it is of some interest to conduct a comparative-dynamic analysis of the effect of a change in the rate of interest. For example, due to an expansive monetary policy, the rate of interest falls. In Figure 1 the  $\dot{K} = 0$  locus remains unchanged and the  $\dot{q} = 0$  locus shifts upward in response to the decline in the rate of interest. Beginning with the steady-state equilibrium,  $E$ , the economy jumps immediately to point  $F$  on the convergent path,  $C'D'$ , and then move to the new steady state,  $E'$ . From this we conclude that a fall in the

rate of interest will stimulate public investment and hence increase the public capital stock.

## (2) Analysis of the Steady-State Equilibrium

The short-run equilibrium given in (10) and (15) defines the derived demands for  $G$ ,  $\tau$ , and  $t$ . In the steady state  $\dot{K} = \dot{q} = 0$ , thus making available two other equations:

$$F(K) = C + \delta K + G, \quad (21a)$$

$$F' + q^{-1}U_K = i + \delta. \quad (21b)$$

Given  $i$  and  $\delta$ , these two equations determine simultaneously the steady-state values of  $K$  and  $q$ .

Equation (21a) expresses that at the steady state the output is divided among private consumption, replacement for depreciation, and governmental consumption expenditures which is just needed to keep  $K$  at its steady-state level. On the other hand, (21b) states that at the optimal level of  $K$ , the marginal benefit of capital, the sum of marginal productivity and the monetary value of marginal utility of capital, equals the rental value, i.e., the interest rate plus the depreciation rate.

Since  $F'$  is the rate of return on the public capital, the steady-state condition (21b) implies that the rate of return should be constant and equal to  $(i + \delta - q^{-1}U_K)$ . On the other hand, if the external benefit of the public capital is also included, then the rate of return is  $(i + \delta)$ .

It may be instructive to compare the steady-state condition for the public capital with that for the private capital. In the latter case  $K$  does not enter as an argument into the utility function and hence  $U_K = 0$ . Equation (21b) is further reduced to  $F' = i + \delta$ .<sup>12</sup> It follows that if other things are equal (i.e., the marginal productivity curve and the depreciation rate of the public capi-

tal are identical with their counterparts of the private capital), the steady-state value of the public capital should at least be equal to that of the private capital.

## **5. Conclusion**

This paper has applied the optimal taxation approach to a simple dynamic model to examine the behaviors of public investment and other fiscal policy instruments at every instant of time. Among other things, we have demonstrated that, when labor supply is assumed to be inelastic, the income tax is in essence identical with a lump-sum tax. It also follows from this assumption that the optimal rate of indirect tax and the level of governmental expenditures should be such that the marginal utility of private consumption is equal to that of governmental consumption. Furthermore, if it is assumed that the marginal productivity curve of the private capital is equal to that of the public capital, then the steady-state value of public capital should be not smaller than the optimal level of private capital.

In this paper, no attention is given to the equity aspect of taxation and public spendings, consistent with our assumption that all individuals in the private sector are identical. The objective of equitable redistribution through public policy can be handled by assuming that the variation in earning capacity among individuals is represented by a variable with finite probability density function [see, for instance, Atkinson and Stiglitz (1980), Lectures 12-14] . As one would expect, the incorporation of randomness into a dynamic model will definitely complicate the analysis.

## **Notes**

1. See Arrow and Kurz (1970), p. 11, for the reason that  $K$  enters as an argument into the utility function.
2. In a paper forthcoming, we shall allow the time preference rate, and



hence private savings, to vary our time.

3. While it would be plausible to allow labor to vary or add the private capital into the production function, neither feature is considered here.
4. The individual is assumed to put all his saving on the public bond.
5. The model can easily be extended to the case a user charge is imposed on the service of the public capital used. In that case, let  $g$  be the per unit service charged, individual's budget constraint is written as  $(1 + \tau)C + gG + B = (1 - t) [F(K) + iB]$ , and the government's budget constraint is  $B = G + I + iB - t [F(K) + iB] - \tau C - gG$ .
6. Equations (7b) and (7c) are also obtained by Turnovsky (1987).
7. Relaxation of this assumption will not affect in a significant way the conclusions we derive later.
8.  $U_{CC} \frac{\partial C}{\partial j} + U_{jC} = -U_{CC} \frac{U_{jC}}{U_{CC}} + U_{jC} = 0, j = G, K$ .
9. This is an alternative way of showing the uniqueness of the solution of the derived demand equations (16a) and (16b).
10. We can derive from (7a) that  $\partial C / \partial \tau = p / U_{CC} (< 0)$  and  $\partial C / \partial j = -U_{Cj} / U_{CC}, j = G, K$ . Therefore,  $\partial C / \partial j = 0$ , as  $U_{Cj} = 0$ .
11. Stability of the system requires that the  $K = 0$  locus cut the  $q = 0$  locus from below. This condition is less strict than the assumption that  $b_{12} < 0$ .
12. This is exactly the equation typically encountered in the literature on private investment. See, for example, Jorgenson (1971).

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# 最適公共投資與財政政策

張慶輝

## 摘 要

雖然公共投資在經濟中所扮演的角色非常重要，學術界對公共部門投資行為的理論研究依然十分缺乏。本文為彌補此項缺陷，特別建立一個簡單的動態模式，並應用最適課稅方法，以檢討公共投資與其他財政變數的最適行為。假設勞動的供給彈性等於零，並設公共投資與課稅的目的，在於追求某一代表個人效用之最大化，並符合政府預算式之限制，本文仔細討論公共資本存量的長期均衡水準，其社會報酬率，及融通此公共計劃所需課徵之所得稅的最適均衡值。