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Quantity Discounted Transportation Rates, Advertising and Industrial Location*

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Abstract

Assuming that a firm uses a single input available from one input source to produce a single output to be sold in the output market, and transportation rates are a function of distance only, Hwang and Mai (1988) recently pointed out that the equilibrium location of the firm is independent of advertising if and only if the production function is homogeneous of degree one. This result is consistent with Sakashita's finding that "demand functions play no role on the location decision of the firm as long as we assume a linear homogeneous production function" (1967:120). However, as is well-known in transportation economics, discounted for quantity, as well as discounted for distance traveled are quite prevalent among the various modes of transportation. This paper incorporates quantity discounts as a key variable into the transportation rate function. By using the unconstrained optimization and the comparative static analysis, this paper shows that the linearly homogeneous production function is not sufficient to insure the independence between advertising and optimum location. This indicates that in general Hwang and Mai's proposition, the optimum location of the firm is independent of advertising if and only if the production function is homogeneous of degree one, can not be applied to the case in which the transportation rate is a function of quantity and distance. It also shows that Hwang and Mai's proposition holds if the elasticities of transportation rate with respect to quantity are constant and identical.

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1. Introduction

Recently Hwang and Mai (1988) investigated the impact of advertising on the location decision of a firm in a linear space. Assuming that a firm uses a single input available from one input source to produce a single output to be sold in the output market, and transportation rates are a function of distance only, Hwang and Mai established the following important proposition.

> Proposition HM. The equilibrium location of the firm is independent of advertising if and only if the production function is homogeneous of degree one. (Hwang and Mai, 1988:224)

They further noted that this proposition is consistent with Sakashita's finding that "demand functions play no role on the location decision of the firm as long as we assume a linear homogeneous production function". (Sakashita, 1967:120). However, as is well-known in transportation economics, discounted for quantity, as well as discounted for distance traveled are quite prevalent among the various modes of transportation. For example, Webber (1984:53-54) pointed out that in 1965, railroads charged virtually the same amount to ship either a five or a ten ton load of paper containers over a short distance. On the other hand, Miller and Jensen (1978), Shieh and Mai (1984), Ziegler (1986), Stahl (1987), and Gilley, Shieh and Williams (1988), and other writers have incorporated quantity discounts into the location theory and examined the implication of quantity discounts on the location decision.

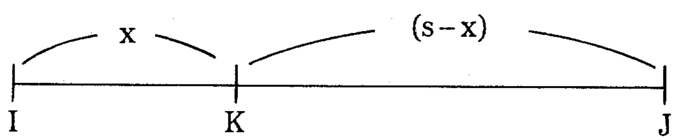
The purpose of this paper is to investigate the effect of quantity discounted transportation rates on the relationship between advertising and location decision of the firm. It shows that Hwang and Mai's proposition needs not hold unless the following condition is included. The elasticities of transportation rates with respect to quantity shipped are constant and identical for input and output.

2. The Model

Following Hwang and Mai (1988), we assume that location is confined to a set of points along a line of length s between I, the market site of a single output, Q, and J, the site of a single input M, as shown in figure 1. x is the distance between I and the firm's location point K, and (s-x) is the distance between K and J. The profit maximizing location problem can be specified as:

$$\pi = [P(Q,A)) - tx]f(M) - [m + r(s-x)]M - cA \tag{1}$$

Figure 1 Location in Linear Space



where $t = x^{ao}Q^{bo}$, $r = (s-x)^{a1}M^{b1}$ are transportation rates of output and input; P(Q,A) is the inverse demand function of the firm at I; m is the given base price of M at its source J, Q is output, A is the level of advertising, and $P_Q < 0$, $P_A > 0$; $Q = f(M)$ with $f_M > 0$ and $f_{MM} < 0$; c is the price per unit of advertising. It should be noted that the inclusion of quantity as an argument in transportation rates constitutes the major departure from Hwang and Mai's model.

Differentiating π with respect to choice variables, x and M yields the first-order conditions for a maximum:

$$\partial\pi/\partial x = -tQ(1+a_0) + rM(1+a_1) = 0 \quad (2)$$

$$\partial\pi/\partial M = [P(.)+p_Q Q]f_M - tx(1+b_0)f_M - m - r(s-x)(1+b_1) = 0 \quad (3)$$

where a_0 and a_1 are elasticities of transportation rate with respect to distance. b_0 and b_1 are elasticities of transportation rate with respect quantity. For simplicity, we assume that a_0, a_1, b_0, b_1 are constant, and $(1+a_0) > 0$, $(1+a_1) > 0$, $(1+b_0) > 0$, $(1+b_1) > 0$.[1] Note that the first term in the right hand side of (2) is the market pull and the second term is the material pull.

Given the second-order conditions, from (2) and (3), we can solve for the optimal values of x and M in terms of A, m, and s:

$$x = x^*(A,m,s) \quad \text{and } M = M^*(A,m,s) \quad (4)$$

where the expressions for the partial derivatives such as $\partial x^*/\partial A$ and $\partial M^*/\partial A$ can be obtained by applying the comparative static analysis.

3. The Impact of Advertising on Location Decision

To investigate the effects of advertising on x* and M*, we differentiate (2) and (3) and apply Cramer's rule to obtain

$$\partial M^*/\partial A = (-1/D)\pi_{MA}\pi_{xx} \quad (5)$$

$$\partial x^*/\partial A = (1/D)\pi_{MAB}\pi_{xM} \quad (6)$$

$$\pi_{MA} = (P_A + P_{QA}Q)f_M \quad (7)$$

$$\pi_{xM} = -(1+a_0)(1+b_0)tf_M + (1+a_1)(1+b_1)r \tag{8}$$

$$\pi_{xx} = -(1+a_0)(\partial t/\partial x)Q - (1+a_1)[\partial r/\partial(s-x)]M \tag{9}$$

$$D = \pi_{xx}\pi_{MM} - (\pi_{xM})^2 \tag{10}$$

where $D > 0$, $\pi_{xx} < 0$, if the second-order conditions are satisfied.

If output and advertising are either complements or independent. $\pi_{MA} > 0$, from (5), we can conclude that[2]

> Proposition 1. An increase in advertising increases the input usage and the sale if output and advertising are either complements or independences.

This result is consistent with Hwang and Mai's Proposition 1. (1988:227) It indicates that Hwang and Mai's Proposition 1 holds even if transportation rates are a function of quantity.

Next, we consider the impact of advertising on the location decision. In general, the sign of $(\partial x^*/\partial A)$ can not a prior be determined, thus the effect of a change in advertising on the location decision is indeterminate. If the production function is homogeneous of degree n, i.e., $f_M M = nQ$. Using (2) in (6) and (8), we obtain.

$$(\partial x^*/\partial A) = -\pi_{MA}(1+a_0)(1+b_1)(tQ/DM)\{[(1+b_0)/(1+b_1)](n-1)\} \tag{11}$$

where $\pi_{MA} > 0$, $t > 0$, $Q > 0$, $M > 0$, $D > 0$, and $(1+a_0) > 0$, $(1+b_0) > 0$, $(1+b_1) > 0$, and, in general, $(1+b_0) \neq (1+b_1)$. It is easy to see that $(\partial x^*/\partial A) \neq 0$, even if $n = 1$. In other words, the linearly homogeneous production function is not sufficient to insure that the optimum location is independent of advertising.

This result is quite different from Hwang and Mai's proposition 2 (1988:228). As is well-known in location theory, the optimum location is found by taking into consideration the relative strength of two forces — the market pull and the material pull. Each pull is comprised of the quantity and marginal transportation cost components. If the production function is linearly homogeneous, when transportation rates are a function of distance only, a change in advertising changes output and input in same proportion. It will not change the relative marginal transport costs of output and input, and then the relative pulls of the market and the material. Thus, the optimum location is independent of advertising. However, when transportation rates are a function of distance and quantity, a change in output and inputs will change the relative marginal transport costs. The relative pulls of the market and material may be affected. Therefore, the linearly homogeneous production function is not sufficient to insure that the optimum location is independent of advertising.

From (11), we can obtain

$$(\partial x^*/\partial A) \lesseqqgtr 0, \text{ if and only if } (1+b_0)n \gtreqqless (1+b_1) \tag{12}$$

where $(1+b_0)n$ = the elasticity of total transport costs of output with respect to the input, $(1+b_1)$ = the elasticity of total transport costs of the input with respect to the input. They describe the impacts of input use on the market pull and the martial pull respectively. Thus, we can conclude

Proposition 2. When the production function is homogeneous of degree n, an increase in advertising will not change the optimum location, if and only if the change of market pull offsets that of material pull, and will move towards (away from) the market, if and only if an increase in market pull is greater (less) than material pull.

Next, we consider that $b_0 = b_1$. The expression of $(\partial x^*/\partial A)$ in (10) can be written as:

$$(\partial x^*/\partial A) = -\pi_{MA}(1+a_0)(1+b_1)(tQ/MD)(n-1) \qquad (13)$$

Since $D > 0$, $(1+a_0) > 0$, $(1+b_0) > 0$ and $\pi_{MA} > 0$, it is easy to show that

$$(\partial x^*/\partial A) \lesseqgtr 0, \text{ if and only if } n \gtreqless 1 \qquad (14)$$

Thus, we have

Proposition 3, When the elasticities of transportation rate with respect to distance are constant, and the elasticities of transportation rate with respect to quantity are identical, an increase in advertising will not change the optimum location if and only if the production function is linearly homogeneous, and will move towards (away from) the market if the production function is increasing (decreasing) returns to scale.

In other words, if the elasticities of transportation rate with respect to quantity are identical, a change in output and input will not change the relative transport costs. Therefore, the impact of advertising on optimum location will be similar to the constant transportation rates case.

4. Concluding Remarks

We have incorporated quantity discounts as a key variable into the transportation rate function and investigated the theoretical implications of this variable on the relationship between advertising and location decision.

Hwang and Mai's study of location theory with the economics of advertising in a linear space focused only on the case where the transportation rate is independent of quantity shipped. Our analysis has generalized the work of Hwang and Mai in the sense that their Proposition 2 can be easily obtained from our model by assuming the elasticities of transportation rate with respect to quantity shipped are zero, or constant and equal.

When the transportation rates are a function of quantity and distance, we have shown that Hwang and Mai's Proposition 1 holds. Furthermore, it is easy to show that Hwang and Mai's propositions 3, 4, and 5 concerning with advertising and social welfare in a linear space hold. We leave it to the interested reader.

Notes

1. It should be noted that $-1 < b_0 < 0$ and $-1 < b_1 < 0$. I owe this point to a thoughful referee.
2. It may be noted that $\pi_{MA} > 0$, if output and advertising are either complement ($P_{QA} > 0$) or independent ($P_{QA} = 0$). In Hwang and Mai (1988:226), they also consider the case in which $\pi_{MA} < 0$, i.e., if output and advertising are strong substitutes [$P_{QA} < 0$ and $-(P_{QA}) > P_A$]. However, $\pi_{MA} < 0$ indicates an increase in advertising reduces marginal revenue. This seems to be unlikely in any model of advertising, at least in the region where advertising is optimally chosen. I owe this point to a thoughtful referee.

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運輸費率對廣告費用和廠商選擇廠址關係的影響

謝勇男

摘要

最近，黃鴻和麥朝成教授很精彩地分析廣告支出對於廠商選擇廠址的影響。假定一個廠商在線型空間上尋找廠址，而運輸費率只和運送路程密切相關。他們發現，如果生產函數是一階齊次式，廠商選擇廠址的決定就不受廣告費用的影響。這一結果和Sakashita 有名的結論不謀而合。Sakashita 認為，如果生產函數是一階齊次式，則需求方面的變動對於廠商選擇廠址的決定沒有任何影響。本文的目的是考慮如果運輸費率也受到運送數量的左右時，黃麥教授的定理在何種情況可以適用。為了比較上的方便，我們只有在黃、麥兩教授的運輸費率函數中加上另外一個變數：運送數量。利用經濟學上通用的最佳選擇法和比較靜態分析，我們發現，即使生產函數是一階齊次式，廠商選擇廠址的決定仍然會受到廣告支出費用影響。這一結果和黃麥定理不同。因為若是運輸費率不受運送數量左右時，廣告支出的起伏，不影響商品引力和原料引力在線型空間的和諧相處。若運輸費率受到運送數量左右時。要使這兩種背道而馳的力量和諧相處，就得費盡九牛二虎之力。結果風吹草動，廣告支出的起伏將影響廠商選擇廠址的決定。不過我們也發現。運輸費率對商品運送數量的彈性和對原料運送數量的彈性若是相對時，這兩種力量將會相安無事，而黃麥兩教授的定理就暢行無阻了。