《人文及社會科學集刊》
第二十五卷第一期（102/3）, pp. 131-144

Cyclical Government Spending and Stabilization under a Balanced Budget Rule*

Shian-jang Tzeng
Doctoral Student
Graduate Institute of Economics, National Taipei University

ABSTRACT

This paper introduces cyclical government spending into a one-sector real business cycle model and systemically examines macroeconomic (in)stability under a balanced budget rule with endogenous government spending (or an endogenous income tax rate). I find that cyclical government spending can stabilize the economy against business cycle fluctuations.

Key Words: cyclical government spending, balanced budget rule, local (in)determinacy, stabilization

I. Introduction

Schmitt-Grohé and Uribe (1997) show that under a balanced-budget rule in which the budget is financed by *endogenous taxes*, it is possible for a steady state to be locally indeterminate, and therefore for sunspot equilibria to occur. By contrast, Guo and Harrison (2004) argue that how the balanced budget rule is implemented is important for indeterminacy; if the tax rate is fixed and the budget is financed by *endogenous government spending*, the steady state can escape from business cycle fluctuations. Such a balanced-budget rule is common in the real business cycle (RBC) literature, e.g., Cooley and Hansen (1992) and Greenwood and Huffman (1991). Nonetheless, Guo and Harrison (2008) further point out that this kind of balanced budget rule still generates sunspot fluctuations in the presence

* I would like to thank Professor Juin-jen Chang, Professor Jhy-yuan Shieh, Professor Tien-wang Tsaur and Chih-hsing Liao for their useful comments and suggestions. All remaining errors are my responsibility.
Received: December 23, 2011; Accepted: July 3, 2012

of productive public services, which improve the productivity of private firms.

However, the functions of government spending not only favor citizens' utility and firms' productivity, but also include cyclical government spending, that is, the government adjusts its expenditure associated with the fluctuations in the real business cycle. In the relevant literature on cyclical government spending, Wagner (1890: 16) provids a formal theoretical framework and finds the covariance of government expenditure and an economy's GDP is positive. This finding is called Wagner's Law (or Wagner's hypothesis). Recently, Ram (1987) used internationally comparable data on GDP and government expenditure for 115 countries to assess the validity of Wagner's Law. He found that the elasticity of government expenditure with respect to GDP is higher than 0.92 for all 115 countries. This evidence confirms that cyclical government spending is a key element of government spending. Indeed, Alesina et al. (2008) also use an approach with political agency to explain the fact that fiscal policy is procyclical in many developing countries. Procyclicality is driven by voters who seek to "starve the Leviathan" to reduce political rents. Voters observe the state of the economy but not the rents appropriated by corrupt governments. When they observe a boom, voters optimally demand more public goods (or larger expenditures), and this induces a procyclical bias in fiscal policy. By contrast, Stekler (1976) provides a comprehensive summary of contracyclical stabilization policy. In the empirical studies, Hercowitz and Strawczynski (2004) emphasize the role of business cycles in the phenomenon of increasing ratios of government spending to GDP in the OECD countries. Their main finding is that the prolonged rise in the ratio of government spending to GDP is partially explained by cyclical upward ratcheting due to asymmetric fiscal behavior: the ratio increases during recessions and is only partially reduced in expansions. Furthermore, Dolls et al. (2012) highlight the importance of assessing the contribution of stable fiscal policy to overall fiscal expansion for the recent economic crisis.

This paper attempts to introduce cyclical government spending into a balanced budget rule with endogenous government spending, and systemically examine whether sunspot fluctuations occur. Our main finding is that, given a balanced budget rule with endogenously productive government spending, the negative relationship between the productive government spending and the income-elastic government spending can protect the economy against business cycle fluctuations. This result has an important policy implication, since the infrastructure provisions have revived the role of government investment as a countercyclical tool in many countries over the past few decades. For example, in response to the recession that began in December 2007, the U.S. Congress passed several fiscal stimulus bills, including the $787 billion American Recovery and Reinvestment Act of 2009. In addition to its large scale, the American Recovery and Reinvestment Act differs from those in the recent past by relying more on spending increases and less on tax cuts. Nearly two thirds of the stimulus package consists of government spending and transfers. Such stimulus spending by governments has been global, commonly appearing in European countries and Australia. Likewise, in Asian countries, such as Japan, Korea, Singapore, and Taiwan, government investment has also been viewed as an important tool for counteracting recessions.

In a recent theoretical study, Stockman (2010) highlights the crucial role of income-elastic government spending by extending the local analysis of Schmitt-Grohé and Uribe (1997) to a *global* one under the balanced budget rule with endogenous tax rates. He finds that the aggregate instability due to a balanced-budget rule suggested by Schmitt-Grohé and Uribe may also be present in the global analysis even though government spending is cyclical to GDP. In an approach that differs from theirs, I not only follow the Guo and Harrison (2004) type of balanced budget rule (which allows for endogenous public spending financed by a fixed income tax rate) but also discuss the Schmitt-Grohé and Uribe (1997) type of balanced budget rule (which allows for an endogenous income tax rate financed by fixed public spending). I further prove that the income-elastic government spending can give rise to a stabilizing effect on the *local* steady-state equilibrium in these different balanced budget rules. This determinacy result contrasts with the notion of Schmitt-Grohé and Uribe (1997) and Guo and Harrison (2008). More importantly, it provides a motivation for policy-makers to implement the commonly-used income-elastic government spending.

II. The analytical framework

Consider an economy which consists of households, firms, and a government. Households produce a single composite commodity that can be consumed and accumulated as capital. Firms produce goods from labor and capital through Cobb-Douglas technology. The government runs a balanced budget rule by endogenously adjusting government spending. It provides infrastructure, which improves the private firms' production, by means of its expenditure. Importantly, I follow Stockman (2010) and specify that government spending is countercyclical in relation to real GDP.

A. Firms

To produce output Y_t, firms rent capital K_t (at the rental rate r_t) and hire labor

H_t (at the wage rate w_t), according to the following Cobb-Douglas technology:

$$Y_t = K_t^{\alpha} H_t^{1-\alpha} \widetilde{G}_t^{\eta}, \tag{1}$$

where $\alpha(1-\alpha)$ is the capital (labor) share and η is the productivity measure of government spending. In line with Barro (1990), the government's public spending $\widetilde{G}_t$ is productive in terms of improving the production.

Given (1), the first-order conditions for the profit maximization of the consumption good producer are:

$$w_t = \frac{(1-\alpha)Y_t}{H_t} \quad \text{and} \quad r_t = \frac{\alpha Y_t}{K_t}. \tag{2}$$

To focus on our point, we impose the restriction $0 < \alpha + \eta < 1$, which ensures the diminishing marginal productivity of a factor.

B. Households

The economy is populated by a unit measure of identical, infinitely-lived households. Each household derives utility from consumption C_t and incurs disutility from labor supply H_t. Specifically, the optimization problem of the representative household is given by:

$$\begin{aligned} &\max_{\{C_t, H_t, K_t\}} U = \int_0^{\infty} [\ln C_t - BH_t] e^{-\rho t} d_t, \quad B > 0, \\ &s.t \;\; \dot{K}_t = (1-\tau)(w_t H_t + r_t K_t) - C_t - \delta K_t, \end{aligned} \tag{3}$$

where $\rho(>0)$ represents the time preference rate, $\tau \in (0, 1)$ is the income tax rate, and $\delta \in (0, 1)$ is the depreciation rate of capital. The specification of utility is characterized by indivisible labor, as argued by Hansen (1985) and Guo and Harrison (2008).[1]

Let λ_t be the co-state variable of the current value Hamiltonian associated with (3). Thus, the necessary optimum conditions for the representative agent are given by:

$$\frac{1}{C_t} = \lambda_t, \tag{4a}$$

$$B = \lambda_t (1-\tau) w_t, \tag{4b}$$

$$\lambda_t [(1-\tau) r_t - \delta] = \lambda_t \rho - \dot{\lambda}_t, \tag{4c}$$

and the transversality condition of $\lim_{t \to \infty} \lambda_t K_t e^{-\rho t} = 0$.

1 This result holds even though I relax the specification of indivisible labor.

C. Government

The government budget constraint is then given by:

$$G_t=\tau(w_tH_t+r_tK_t). \tag{5}$$

The fiscal authorities adhere to the balanced budget rule: endogenize public spending financed by a fixed tax rate on income. While such a balanced budget rule contrasts with that of Schmitt-Grohé and Uribe (1997), it is commonly used in the RBC literature, as argued by Guo and Harrison (2004).

With particular emphasis, government spending is not only endogenous, but also cyclical in relation to output Y_t. To be more specific, by following Stockman (2010), government spending is specified as follows:

$$G_t=\tilde{G}_t\left(\frac{Y_t}{\hat{Y}}\right)^{\gamma}, \tag{6}$$

where $\gamma\in(-1, 1)$. Equation (6) indicates that government spending consists of two parts: one is the autonomous government expenditures which favor private production, i.e., $\tilde{G}_t$, and the other reflects the cyclical government spending (or equivalent to the income-elastic government spending used by Schmitt-Grohé and Uribe (1997) and Stockman (2010)), i.e., $(Y_t/\hat{Y})^{\gamma}$; here Y_t denotes the actual level of GDP, $\hat{Y}$ is the steady-state level of GDP and $\gamma(=d\ \ln(Y_t/\hat{Y})^{\gamma}/d\ \ln\ Y_t)$ is the income elasticity of the cyclical government spending. Given (5) and (6), the income-elastic government spending $(Y_t/\hat{Y})^{\gamma}$ makes the balanced budget rule with endogenous public spending $\tilde{G}_t=\tau Y_t(Y_t/\hat{Y})^{-\gamma}$. The negative relationship between productive government expenditures and income-elastic government spending can be clearly shown. To examine the robustness of the underlying result, we define the income elasticity of the balanced budget rule with endogenous public spending as: $\theta_G\equiv d\ \ln\ \tilde{G}_t/d\ \ln\ Y_t$. By the definition of θ_G and the balanced budget rule with endogenous public spending, we can derive this θ_G as: $\theta_G=1-\gamma$.

D. Equilibrium

Putting the budget constraints of the household (3) and of the government (5) together leads to the aggregate resource constraint:

$$\dot{K}_t=Y_t-C_t-G_t-\delta K_t, \tag{7}$$

which is essentially the good market-clearing condition. In addition to (7), by using (2), (4a)-(4c), and (6), the economy's equilibrium can be summarized as:

$$\frac{\dot{C}_t}{C_t}=\frac{\alpha(1-\tau)Y_t}{K_t}-\delta-\rho, \tag{8}$$

$$H_t=\frac{(1-\tau)(1-\alpha)Y_t}{BC_t}, \tag{9}$$

$$\widetilde{G}_t=\tau Y_t\left(\frac{Y_t}{\hat{Y}}\right)^{-\gamma}. \tag{10}$$

Equation (8) is the Euler equation of consumption, (9) is the equilibrium condition of the labor market, and (10) characterizes the balanced budget rule with endogenous productive government spending.

III. Cyclical government spending and stabilization

To derive the log-linear dynamical system under the balanced budget rule with endogenous government spending, we define the following logarithmic transformation of variables as: $y_t\equiv\log Y_t$, $k_t\equiv\log K_t$ and $c_t\equiv\log C_t$. By using these logarithmic transformations of variables and taking the logarithm with (1), (9) and (10) and manipulating the resulting equations, we can obtain:

$$y_t-k_t=\varphi_1 c_t+\varphi_2 k_t+\Psi, \tag{11}$$

where $\varphi_1=\frac{(\alpha-1)}{[\alpha-\eta(1-\gamma)]}$, $\varphi_2=\frac{\eta(1-\gamma)}{[\alpha-\eta(1-\gamma)]}$ and $\Psi=\frac{(1-\alpha)\ln\left[\frac{(1-\tau)(1-\alpha)}{B}\right]+\eta(\ln\tau+\gamma\hat{y})}{[\alpha-\eta(1-\gamma)]}$.

Given (11), we log-linearize the aggregate resource constraint (7) and the Euler equation of consumption (8) and, accordingly, further have:

$$\dot{c}_t=\alpha(1-\tau)\exp^{(y_t-k_t)}-\delta-\rho,$$
$$\dot{k}_t=(1-\tau)\exp^{(y_t-k_t)}-\exp^{(c_t-k_t)}-\delta,$$

then substituting (11) into the above dynamic system as:

$$\dot{c}_t=\alpha(1-\tau)\exp^{(\varphi_1 c_t+\varphi_2 k_t+\Psi)}-\delta-\rho, \tag{12}$$
$$\dot{k}_t=(1-\tau)\exp^{(\varphi_1 c_t+\varphi_2 k_t+\Psi)}-\exp^{(c_t-k_t)}-\delta. \tag{13}$$

Equations (12) and (13) constitute a 2×2 dynamic system in terms of c_t and k_t.

A. Local (in)determinacy

Based on (12) and (13), I establish:

Proposition 1 (stabilization policy). *Given $\gamma\in(0,1)$, if the balanced budget rule allows for endogenous public spending financed by a fixed income tax rate and* *further generates the income elasticity of the balanced budget rule with endogenous public spending* $\theta_G \in (0,1)$, *the necessary and sufficient condition for local determinacy is described as*:

$$\alpha > \eta\theta_G.$$

Proof: To complete this, I first characterize the properties of the steady state of economy. In the steady state: $\dot{c}_t = \dot{k}_t = 0$ and let $c_t = \hat{c}$ and $k_t = \hat{k}$ as the characteristics of the steady state of c_t and k_t. The steady state levels of $\hat{c}$, $\hat{k}$, and $\hat{y}$ are easily obtained from (11), (12), and (13):

$$\hat{c} = \frac{1}{1-(\alpha+\eta)}\left\{\eta \ln \tau + (1-\alpha)\ln\left[\frac{(1-\alpha)(\delta+\rho)}{B\delta(1-\alpha)+B\rho}\right] - (1-\eta)\ln\left[\frac{\delta+\rho}{\alpha(1-\tau)}\right]\right\} + \ln\frac{[(1-\alpha)\delta+\rho]}{\alpha}, \quad (14)$$

$$\hat{k} = \frac{1}{1-(\alpha+\eta)}\left\{\eta \ln \tau + (1-\alpha)\ln\left[\frac{(1-\alpha)(\delta+\rho)}{B\delta(1-\alpha)+B\rho}\right] - (1-\eta)\ln\left[\frac{\delta+\rho}{\alpha(1-\tau)}\right]\right\}, \quad (15)$$

$$\hat{y} = \frac{1}{1-(\alpha+\eta)}\left\{\eta \ln \tau + (1-\alpha)\ln\left[\frac{(1-\alpha)(\delta+\rho)}{B\delta(1-\alpha)+B\rho}\right] - \alpha\ln\left[\frac{\delta+\rho}{\alpha(1-\tau)}\right]\right\}. \quad (16)$$

Equcations (14)–(16) show that my model exhibits a unique interior steady state. By using equations (14)–(16) and computing the Jacobian matrix of (12) and (13) evaluated at the steady state, I can describe the following conditions of trace and determinant of the Jacobian matrix:

$$Trace(J) = \frac{\alpha\rho + \eta\theta_G\delta}{\alpha - \eta\theta_G} \gtreqless 0 \ \text{ if } \ \alpha \gtreqless \eta\theta_G, \quad (17)$$

$$Det(J) = \frac{\Theta}{\alpha - \eta\theta_G} \gtreqless 0 \ \text{ if } \ \alpha \lesseqgtr \eta\theta_G, \quad (18)$$

where $\Theta = (\delta+\rho)[(1-\alpha)\delta+\rho][(\alpha+\eta)-1-\eta\gamma] < 0$. In this dynamic system there is only one jump variable c_t. Thus (14) and (15) show that there exists a unique perfect-foresight equilibrium path (saddle-path stability) if two eigenvalues are of opposite sign; however, there exists a continuum of equilibrium trajectories that converges to the steady state (sink) if the two eigenvalues are negative. That is, local determinacy (indeterminacy) occurs if $\alpha > \eta\theta_G (\alpha < \eta\theta_G)$.

If government spending is useless, i.e., $\eta = 0$, $\alpha > \eta\theta_G$ automatically holds and, as a result, local determinacy occurs. This case recovers the result of Guo and Harrison (2004). By contrast, if government spending is productive ($\eta > 0$) and cyclical government spending is ignored ($\gamma = 0 \Rightarrow \theta_G = 1$), the necessary and sufficient condition for generating local indeterminacy becomes $\alpha < \eta$, as shown in Guo and Harrison (2008). Given that the equilibrium wage-hours locus becomes upward sloping, and steeper than the labor supply curve, the Guo–Harrison model with productive public expenditures is qualitatively equivalent to Benhabib and Farmer's (1994) laissez-faire economy with aggregate increasing returns-to-scale in production. Under their model, this steady-state indeterminacy seems to be possible empirically. Given that $\alpha=0.33$ and $\eta=0.39$ (estimated using U.S. data, Aschauer, 1989), $\alpha<\eta$ holds true empirically. Therefore, the balanced budget rule, in which the tax rate is fixed and the budget is financed by adjusting government spending, generates sunspot fluctuations as government spending is useful to the private production.

Proposition 1 clearly demonstrates that the cyclical government spending with $\gamma\in(0,1)$ which favors private production can stabilize the economy against sunspot fluctuations caused by the balanced budget rule with productive government spending. This result suggests that if productive government spending is cyclical to GDP and the positive income-elastic government spending γ is crucial to make $\alpha-\eta\theta_G>0$ (or equivalent to $\alpha-\eta(1-\gamma)>0$), the balanced budget rule will result in local determinacy, rather than indeterminacy. Using U.S. data of $\alpha=0.33$, $\eta=0.39$, and the necessary and sufficient condition for local determinacy: $\alpha-\eta\theta_G>0$, I can find that the possible range of γ is $\gamma>0.154$. Actually, all possible numerical values of γ are summarized by the following table.

Table 1

Schmitt-Grohé and Uribe (1997); Stockman (2010)	$\gamma=0.5$
Ram (1987)	$0.92<\gamma<0.991$
Kolluri, et al. (2000)	$0.27<\gamma<0.487$

Obviously, these values of the propensity of government spending with the income-elastic government spending γ are more than the critical value of $\gamma(\gamma=0.154)$ as showed by Table 1. Therefore, the cyclical government spending with $\gamma\in(0,1)$ which favors private production will stabilize the economy against sunspot fluctuations.

B. Interpretation

In order to glean the intuition for local indeterminacy, I describe the Euler equation (8) and the equilibrium condition of the labor market (9) in the following discrete-time manner, respectively:

$$\frac{C_{t+1}}{C_t}=\beta[(1-\tau)r_{t+1}+(1-\delta)]=\beta\left[\frac{\alpha(1-\tau)Y_{t+1}}{K_{t+1}}+(1-\delta)\right], \tag{19}$$

$$H_{t+1}=\frac{(1-\tau)(1-\alpha)Y_{t+1}}{BC_{t+1}}. \tag{20}$$

Rewrite (1) by the discrete-time manner and reuse (20) and, accordingly, I further have: $r_{t+1}=\alpha\psi[C_{t+1}^{\alpha-1}/K_{t+1}^{\eta\theta_G}]^{1/(\alpha-\eta\theta_G)}$, $\psi=\alpha\{(\tau\hat{Y})^{\eta}[(1-\tau)(1-\alpha)/B]^{1-\alpha}\}^{1/(\alpha-\eta\theta_G)}>0$. We then substitute $r_{t+1}=\alpha\psi[C_{t+1}^{\alpha-1}/K_{t+1}^{\eta\theta_G}]^{1/(\alpha-\eta\theta_G)}$ into (19) as:

$$\frac{C_{t+1}}{C_t}=\beta\Bigg\{(1-\tau)\underbrace{\alpha\psi\left[\frac{C_{t+1}^{\alpha-1}}{K_{t+1}^{\eta\theta_G}}\right]^{\frac{1}{\alpha-\eta\theta_G}}}_{r_{t+1}}+(1-\delta)\Bigg\}, \tag{21}$$

equation (21) is the discrete-time manner of the Euler equation of consumption; where $\beta=1/(1+\rho)>0$. This discrete-time manner of the Euler equation of consumption can help us to illustrate the intuition of Proposition 1. Suppose that agents become optimistic about the future return on capital, i.e., r_{t+1}. In acting upon this belief, the household will sacrifice consumption today (C_t decreases below its steady-state level) for more investment (and hence K_{t+1} increases). This implies that the level of the future output Y_{t+1} and labor hours H_{t+1} will increase, and the future consumption C_{t+1} will increase as well. A lower C_t and a higher C_{t+1} indicate that the value of the LHS of (21) increases because of households' optimistic expectations. In order to remain in equilibrium, the RHS of (21) must also increase.

By focusing on the Guo and Harrison (2008) case ($\eta>0$ and $\gamma=0$), higher future consumption C_{t+1} and capital K_{t+1} raise the value of the RHS of (21), provided that the condition $\alpha<\eta$ holds true. Under such a situation, as shown in (21), agents' expectations will be self-fulfilling. Intuitively, productive public services will raise the marginal productivity of capital in the private sector (and hence the future return on capital r_{t+1}) and, accordingly, help agents' optimistic expectations to become self-fulfilling. Thus, sunspot fluctuations occur.

To gain the stabilizing effect for the cyclical government spending with $\gamma\in(0,1)$, the economy must exhibit saddle path stability (local determinacy), which requires that $\alpha-\eta\theta_G>0$. As noted above, if the income-elastic government spending $\gamma>0.154$, the role of income-elastic government spending can overturn the Guo and Harrison (2008) condition and, as a result, remove indeterminacy. In the presence of the cyclical government spending with $\gamma\in(0,1)$, households are aware that when output increases above its steady-state level, the government will reduce its spending on infrastructure, which is unfavorable to the future return on capital r_{t+1}. Equation (21) indicates that the decrease in r_{t+1} contradicts the intertemporal Euler equation, and this contradiction invalidates the initial rise in the expected return on capital. As a result, cyclical government spending with $\gamma\in(0,1)$ prohibits agents' optimistic expectations from being self-fulfilling and stabilizes the economy against sunspot fluctuations.

IV. Discussion

Several assumptions in Section 3 are debatable. Therefore, in this section, I will relax them and provide extensive discussion accordingly.

A. The additive form of government spending

To test the robustness of Proposition 1 that is held by the multiplicative form of government spending, we further relax the setting of (6) as the following additive form:

$$G = G^a + G^c = \widetilde{G}_t + G_0^c\left(\frac{Y_t}{\hat{Y}}\right)^{\gamma}, \tag{22}$$

where $G^a = \widetilde{G}_t$ is the government expenditures which favor private production, $G^c = G_0^c(Y_t/\hat{Y})^{\gamma}$ is the induced government spending and G_0^c is the constant parameter. By (5) and (22), we obtain:

$$\widetilde{G}_t = \tau Y_t - G_0^c\left(\frac{Y_t}{\hat{Y}}\right)^{\gamma}, \tag{23}$$

and accordingly derive the income elasticity of the balanced budget rule with endogenous public spending from (23) as:

$$\widetilde{\theta}_G = \frac{\tau \exp^{\hat{y}} - \gamma G_0^c}{\tau \exp^{\hat{y}} - G_0^c} > 1, \tag{24}$$

where $\hat{y}$ is independent for γ. Equation (24) indicates that the income elasticity of the balanced budget rule with endogenous public spending is always higher than unity regardless of whether $\gamma \in (-1, 1)$. Given (24), the necessary and sufficient condition for local determinacy is described by:

$$\alpha > 1 - \eta\left(\frac{\tau \exp^{\hat{y}} - \gamma G_0^c}{\tau \exp^{\hat{y}} - G_0^c}\right). \tag{25}$$

Because $\widetilde{\theta}_G > 1$ always holds under this additive form, it is easier to satisfy the result of local determinacy (25) while the productivity measure of government spending η is high enough and such that $1 - \eta\widetilde{\theta}_G < 0$.

B. The balanced budget rule with endogenous income tax rate ($\tilde{G}t=G$)

Schmitt-Grohé and Uribe (1997) illustrate that a balanced-budget rule with an endogenous income tax rate can lead to aggregate instability. In particular, under such a rule it is possible for a steady state to be locally indeterminate, and therefore sunspot equilibria are possible. In this case, I would like to test whether the result of Schmitt-Grohé and Uribe (1997) still holds if total government spending includes the income-elastic government spending. To gain the stabilizing effect for the income-elastic government spending, we rewrite the balanced budget rule with endogenous income tax rate as:

$$\tau_t=\frac{G}{Y_t}\left(\frac{Y_t}{\hat{Y}}\right)^{\gamma}, \tag{26}$$

where G is the fixed government expenditures which favor private production. We also define the income elasticity of the balanced budget rule with endogenous income tax rate as:

$$\theta_\tau\equiv-\frac{d\ln\tau_t}{d\ln Y_t}, \tag{27}$$

and calculate the result of θ_τ from (26) and (27) as:

$$\theta_\tau=1-\gamma. \tag{28}$$

Given (28), the necessary and sufficient condition for local determinacy is described by:

$$\theta_\tau<\frac{\alpha(1-\hat{\tau})}{\hat{\tau}(1-\alpha)}, \tag{29}$$

where $\hat{\tau}=(G/\exp^{-\hat{y}})\in(0,1)$. The intuition of (29) is that higher γ can lower θ_τ, thus reducing the after-tax marginal productivity of capital in the private sector (and hence the future return on capital $(1-\tau_{t+1})r_{t+1}$), as a result, the cyclical government spending with higher γ prevents agents' optimistic expectations from being self-fulfilling and stabilizes the economy against sunspot fluctuations. Therefore, the main result of Proposition 1 is still valid.

V. Conclusion

This paper has systematically examined the stabilizing effect of cyclical government spending when a government follows a balanced budget rule. In contrast to Guo and Harrison (2008) (or Schmitt-Grohé and Uribe (1997)), I have shown that the balanced budget rule with productive government spending (or income tax rate) may result in local determinacy, rather than indeterminacy. This result implies that cyclical government spending can stabilize the economy against business cycle fluctuations. It then provides an explanation for the motivation of the commonly implemented cyclical government spending.

References

Alesina, A., F. R. Campante, and G. Tabellini
2008 "Why Is Fiscal Policy Often Procyclical?" *Journal of the European Economic Association* 6(5): 1006-1036.

Aschauer, D. A.
1989 "Is Public Expenditure Productive?" *Journal of Monetary Economics* 23(2): 177-200.

Barro, R. J.
1990 "Government Spending in a Simple Model of Endogenous Growth," *Journal of Political Economy* 98(5): 103-125.

Benhabib, J. and R. E. A. Farmer
1994 "Indeterminacy and Increasing Returns," *Journal of Economic Theory* 63(1): 19-41.

Cooley, T. F. and G. Hansen
1992 "Tax Distortions in a Neoclassical Monetary Economy," *Journal of Economic Theory* 58(2): 290-316.

Dolls, M., C. Fuest, and A. Peichl
2012 "Automatic Stabilizers and Economic Crisis: US vs. Europe," *Journal of Public Economics* 96(3): 279-294.

Greenwood, J. and G. W. Huffman
1991 "Tax Analysis in a Real-business-cycle Model: On Measuring Harberger Triangles and Okun Gaps," *Journal of Monetary Economics* 27(2): 167-190.

Guo, J.-T. and S. Harrison
2004 "Balanced-budget Rules and Macroeconomic (In)stability," *Journal of Economic Theory* 119(2): 357-363.
2008 "Useful Government Spending and Macroeconomic (In)stability under Balanced-budget Rules," *Journal of Public Economic Theory* 10(3): 383-397.

Hansen, G.
1985 "Indivisible Labor and the Business Cycle," *Journal of Monetary Economics* 16(3): 309-327.

Hercowitz, Z. and M. Strawczynski
2004 "Cyclical Ratcheting in Government Spending: Evidence from the OECD," *Review*

of Economics and Statistics 86(1): 353–361.

Kolluri, B., M. Panik, and M. Wahab

2000 "Government Expenditure and Economic Growth: Evidence from G7 Countries," *Applied Economics* 32(8): 1059–1068.

Ram, R.

1987 "Wagner's Hypothesis in Time-series and Cross-section Perspectives: Evidence from 'Real' Data for 115 Countries," *Review of Economics and Statistics* 69(2): 194–204.

Schmitt-Grohé, S. and M. Uribe

1997 "Balanced-budget Rules, Distortionary Taxes, and Aggregate Instability," *Journal of Political Economy* 105(5): 976–1000.

Stekler, H. O.

1976 "Economic Forecasting and Contracyclical Stabilization Policy," *Journal of Public Economics* 5(3/4): 225–236.

Stockman, D. R.

2010 "Balanced-budget Rules: Chaos and Deterministic Sunspots," *Journal of Economic Theory* 145(3): 1060–1085.

Wagner, A.

1890 *Finanzwissenschaft*. 3rd Edition. Leipzig: C. F. Winter.

平衡預算法則下的循環性政府支出與均衡動態穩定

曾憲政
國立臺北大學經濟學系博士生

摘　　要

本文引進循環性政府支出至一個單部門的實質景氣循環模型，且系統地檢視財政當局執行「內生調整政府支出（或是內生調整所得稅率）的平衡預算法則」的總體經濟穩定特質。我們發現：循環性政府支出的引進可以穩定體系自我信念驅動的景氣波動。

關鍵字：循環性政府支出、平衡預算法則、局域（非）唯一解、穩定性