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Where to Locate in a Circular City with a Foreign Market?*

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ABSTRACT

This study considers a spatial Cournot competition between duopoly firms in a circular market with an exporting point connected to a foreign market. It is shown that the relative size of the foreign market is crucial in determining the location equilibrium. Specifically, when the foreign market is small, there exists a separated location equilibrium. As the foreign market size increases, the separated equilibrium locations move closer to the exporting point. When the foreign market size is relatively large, both firms agglomerate at the exporting point. Our results are robust in the case of mixed duopoly. Moreover, the equilibrium locations are either farther apart or closer to each other than the socially optimal locations for the domestic country. In addition, we explore several extensions such as two circular markets, export subsidies, and the competition between a domestic firm and a foreign firm. Finally, implications for

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investment promotion are also provided.

Key Words: locations, spatial Cournot model, circular market, export subsidies, foreign market

I. Introduction

This study presents a novel circular city (market) model with domestic duopolists and an exporting point connected to a foreign market.[1] We contribute to the current spatial competition literature by including the influence of a foreign market, which is essential in the modern economy. We emphasize the influence of the relative size of the foreign market to the domestic market on firms' location choices. For example, many manufacturing firms in China, a giant exporting country, have chosen to locate at the coastal regions in order to lower exporting costs since the 1980s. Recently, some of these firms have moved to inland regions after the domestic demand increase (for example, see Sun, 2001).

Concerns over the role of elastic demand in the spatial model have motivated several studies on Cournot competition. Hamilton et al. (1989) and Anderson and Neven (1991) initially develop a spatial Cournot model in which duopoly firms engage in a location competition and then quantity competition in a linear city model. They show that duopolists agglomerate at the market center in equilibrium. Pal (1998) further establishes a circular city (market) framework and shows that duopolistic firms locate at the two ends of a diameter in equilibrium. His model is extended to allowing an even number of firms by Matsushima (2001), who shows that half of firms locate at one end of a diameter and the other half locate at the other end of this diameter in an equilibrium. That is, the equilibrium locations in a circular market can be partially agglomerated and partially separated. Recently, both Sun (2010) and Ago (2013) show that agglomeration can appear in equilibrium under different mechanisms. Specifically, the duopoly firms will agglomerate at the same point when they deliver their products in different directions, while an equidistant location equilibrium is the unique result when both firms deliver their products in the same direction. Ago (2013) considers an additional mass of mobile

1 Involving foreign trade in spatial models has been examined in Schmitt (1995), which analyzes the influence of trade barriers on product differentiation in a traditional Hotelling model (with inelastic demand). Furthermore, Tharakan and Thisse (2002) consider a Hotelling duopoly model of international trade in which the two countries have a common border and the locations of firms are exogeneous. They show that the small country always gains from trade, while the large country always loses from trade.

consumers attracted by each duopolist, and shows that two firms will agglomerate if the mass of mobile consumers is large, and appear to be in-between differentiation if such a mass is moderate.

Gupta et al. (2004) provide a generalized framework that includes all the above models, and find many possible location patterns in a circular market with multiple firms.[2] Gupta et al. (2006) further analyze the scenario in which firms produce substitutes and/or complements with linear, convex, and concave transport costs, and present multiple location equilibria. Recently, Matsumura and Matsushima (2012) introduce non-linear transportation cost functions and show that none of the asymmetric location patterns obtained by Gupta et al. (2004) could be equilibrium outcomes. Matsumura and Shimizu (2005) analyze the socially optimal locations in a linear city model, and show that the distance between two equilibrium locations is always shorter than the socially desirable distance.

Our study finds that the relative size of the foreign market is crucial for the location choices of firms. Specifically, when the size of the foreign market is small, there exists a dispersed location equilibrium. As the foreign market size increases, both firms move closer to the exporting point, in order to save on transportation costs to the foreign market. When the size of the foreign market converges to nil, the duopoly firms will focus mostly on the domestic market, and so the equilibrium locations should converge to the endpoints of a diameter, and our model is thus degenerated to Pal (1998). As the size of the foreign market grows to a certain level, firms choose to agglomerate at the exporting point.

This study also derives the socially optimal locations. We show that the equilibrium locations are either farther apart or closer to each other than the socially

2 Other related models are as follows. Chamorro-Rivas (2000) revises Pal (1998) by allowing each firm to have two stores, and shows that each firm will set its two stores at the two ends of a diameter and that these two diameters are perpendicular to each other in equilibrium. Shimizu (2002) considers the characteristics (substitute or complementary) of the products of the two firms, and shows that both firms will agglomerate at a single point when their products are complements. His result is the opposite of that of Pal (1998). Yu and Lai (2003) generalize these two models and show that each firm sets its stores at the two ends of a diameter and that these diameters are perpendicular (coincident) when products are substitutes (complements). Recently, Ebina et al. (2011) adopt a general model to contain both linear and circular markets as their special cases. They assume a point in a circular market exists such that transporting goods across that point adds an additional cost of $\beta \in [0, 1]$. Therefore, the market is a circular market when $\beta = 0$, but linear when nothing can be transported across this point as β is large. They show that the equilibrium location is discontinuous with respect to β, and multiple equilibria exist for a certain range of β.

optimal locations for the domestic country. In addition, our results are robust in the case of mixed duopoly, which has been discussed in Matsushima and Matsumura (2003; 2006).[3]

In addition to one circular market connected with a foreign market, we also explore an extension of two asymmetric circular markets. When the foreign circular market is relatively large, two domestic firms agglomerate at the exporting point, while when the foreign circular market is relatively small, a pair of dispersed locations constitute the equilibrium. Moreover, it will be shown that an export subsidy might affect the location choices of firms. Intuitively, an export subsidy reduces the transport costs of export, and thus firms tend to locate closer to the exporting point. Furthermore, there exist multiple types of corner equilibria in the case of one domestic firm and one foreign firm.

The rest of this article is organized as follows. Section 2 presents the model and Section 3 discusses the equilibrium locations. Section 4 analyzes the implications of socially optimal locations for the domestic firms. Section 5 discusses the scenario of mixed duopoly. Section 6 provides several extensions to highlight the value of the circular market model with a foreign market. Section 7 draws some concluding remarks.

II. The model

Consider one circular city (market) with one unit length which is connected to a foreign market via an exporting port at point zero (see Fig. 1). For simplicity, assume that the foreign market is represented by a point, and the distance between the exporting point and the foreign market is K, $K \geq 0$.[4] There is a mass of representative consumers who are uniformly distributed over all points $x \in [0, 1]$. The utility function of the consumer x is $u(q(x)) = m + a \cdot q(x) - b \cdot q(x)^2/2$, where m is the consumption of the numeraire, and $q(x)$ is the product quantity. The budget constraint requires $I = m + p(x) \cdot q(x)$, where I is a constant and identical endowment and $p(x)$ is the market price. Utility maximization yields the inverse demand function at x:

3 They discuss mixed oligopoly in a circular domestic market, and show that the private firms will agglomerate at the point remotest from (opposite to) the public firm (see Matsushima and Matsumura, 2003). The case with foreign firms is further included in Matsushima and Matsumura (2006).

4 When K is large, the foreign market can be treated as a point market, while the spatial configuration of the foreign market for cases when K is not large will be discussed later in Section 6.

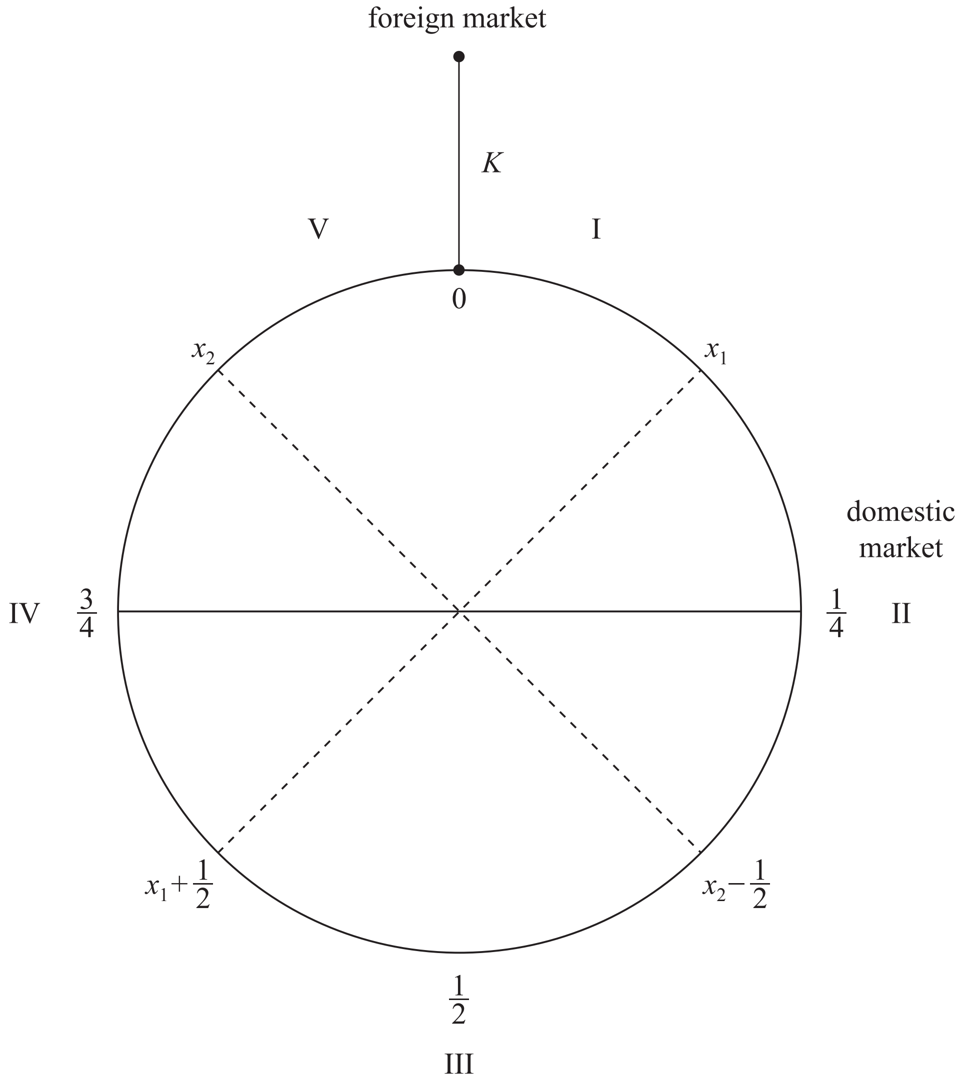

Fig. 1. The configuration of the domestic market and a foreign market

$$p(x)=a-bq(x)=a-b(q_1(x)+q_2(x)), \tag{1}$$

where $q_i(x)$ is the quantity that is supplied by firm i at x, $i=1, 2$, $q(x)$ is the total quantity at x, a is the reservation price, and b is the absolute value of the slope of the demand function. Suppose there are F representative consumers in the foreign market. The demand function of the foreign market is $q^f=F \cdot q=[a/b-1/b \cdot p^f] \cdot F$, where the superscript "f" represents the foreign market, and so the inverse demand function of the foreign market is

$$p^f=a-\frac{b}{F}q^f. \tag{2}$$

Suppose that two identical firms (1 and 2) engage in a two-stage spatial Cournot competition with zero production costs. In the first stage, both firms select their locations (x_1 and x_2) simultaneously, where $x_1 \in [0, 1/2]$, $x_2 \in [1/2, 1]$, and $|x_2 - x_1| \geq 1/2$ without loss of generality.[5] We define the boundary locations as $x_1 = 0$ or 1/2, $x_2 = 1/2$ or 1. The location pairs $x_1 \in (0, 1/2)$ and $x_2 \in (1/2, 1)$ are represented as interior locations. In the second stage, firms simultaneously determine their quantities at each point of the circular market. For each location $x \in [0, 1]$, the profit functions of firms are

$$\pi_i(x) = (a - b(q_1(x) + q_2(x)) - t \cdot d(x, x_i))q_i(x), \quad i = 1, 2, \tag{3}$$

where t is the transport rate in the domestic market, and $d(x, x_i) = \min\{|x - x_i|, 1 - x_i + x\}$ is the minimal distance between x and x_i on the circular market. Assume that a is sufficiently large such that $a \geq t(1+K)$ to ensure that all the market points are served by both firms.[6] Simultaneously solving $\partial \pi_i(x)/\partial q_i = 0$, $i = 1, 2$ in the second stage yields

$$q_1 = \frac{1}{3}\frac{a + td(x, x_2) - 2td(x, x_1)}{b}, \quad q_2 = \frac{1}{3}\frac{a - 2td(x, x_2) + td(x, x_1)}{b}. \tag{4}$$

So, the equilibrium prices are

$$p(x) = \frac{1}{3}(a + td(x, x_1) + td(x, x_2)), \quad x \in [0, 1]. \tag{5}$$

Similarly, for the foreign market, the profits functions for the two domestic firms are

$$\pi_i^f = \left(a - \frac{b}{F}(q_1^f + q_2^f) - t(d(0, x_i) + K)\right)q_i^f, \quad i = 1, 2. \tag{6}$$

Then, the equilibrium quantities are

$$q_1^f = F \cdot \frac{a - tK + t - tx_2 - 2tx_1}{3b}, \quad q_2^f = F \cdot \frac{a - tK - 2t + 2tx_2 + tx_1}{3b}, \tag{7}$$

and the equilibrium foreign price in the second-stage subgame is

$$p^f = \frac{1}{3}(a + 2tK + t - tx_1 - tx_2). \tag{8}$$

5 It is noted that locations satisfying $|x_2 - x_1| \geq \frac{1}{2}$ are dominant strategies, because there exists a centripetal force from the foreign market which is connected through the exporting point ($x = 0$).

6 This condition can be derived by substituting $x_2 = \frac{1}{2}$, the remotest location of firm 2, and $x_1 = 0$, the nearest location of firm 1, into the equilibrium quantities $q_i^f \geq 0$, $i = 1, 2$. We are grateful to one of the referees for offering this stringently sufficient condition.

We divide the circular market into five segments in Fig. 1 for further calculations: region I is $[0, x_1]$, region II is $[x_1, x_2-1/2]$, region III is $[x_2-1/2, x_1+1/2]$, region IV is $[x_1+1/2, x_2]$, and region V is $[x_2, 1]$. The total profits are the sum of profits from these five segments and from the foreign market:[7]

$$\Pi_1=\int_0^{x_1}\frac{(t-tx_2+3tx+a-2tx_1)^2}{9b}dx+\int_{x_1}^{x_2-\frac{1}{2}}\frac{(t-tx_2-tx+a+2tx_1)^2}{9b}dx$$
$$+\int_{x_2-\frac{1}{2}}^{x_1+\frac{1}{2}}\frac{(tx_2-3tx+a+2tx_1)^2}{9b}dx+\int_{x_1+\frac{1}{2}}^{x_2}\frac{(tx_2+tx+a-2t-2tx_1)^2}{9b}dx$$
$$+\int_{x_2}^{1}\frac{(-tx_2+3tx+a-2t-2tx_1)^2}{9b}dx+\pi_1^f, \tag{9}$$

$$\Pi_2=\int_0^{x_1}\frac{(-2t+2tx_2-3tx+a+tx_1)^2}{9b}dx+\int_{x_1}^{x_2-\frac{1}{2}}\frac{(-2t+2tx_2-tx+a-tx_1)^2}{9b}dx$$
$$+\int_{x_2-\frac{1}{2}}^{x_1+\frac{1}{2}}\frac{(-2tx_2+3tx+a-tx_1)^2}{9b}dx+\int_{x_1+\frac{1}{2}}^{x_2}\frac{(-2tx_2+tx+a+t+tx_1)^2}{9b}dx$$
$$+\int_{x_2}^{1}\frac{(2tx_2-3tx+a+t+tx_1)^2}{9b}dx+\pi_2^f. \tag{10}$$

In the next section the equilibrium locations in the first stage will be analyzed.

III. Equilibrium locations

Back to the first stage, we first examine the interior (separated) solutions. Solving $\partial\Pi_1/\partial x_1=0$ and $\partial\Pi_2/\partial x_2=0$ simultaneously yields equilibrium locations x_1 and x_2. We then check the boundary locations. In fact, when F is very small such that $t^2(1+2F)^2-8Ft(\alpha-tK)>0$, there indeed exist two asymmetric locations (one interior and one boundary): $(x_1, x_2)=\left(\frac{1}{4t}\left(t(1+2F)+\sqrt{t^2(1+2F)^2-8Ft(a-tK)}\right), 1\right)$, with $\partial x_1/\partial F<0$, $\partial x_1/\partial t>0$, and $\partial x_1/\partial K>0$ and $(x_1, x_2)=\left(0, \frac{1}{4t}\left(t(3-2F)-\sqrt{t^2(1+2F)^2-8Ft(a-tK)}\right)\right)$, with $\partial x_2/\partial F>0$, $\partial x_2/\partial t<0$, and $\partial x_2/\partial K<0$. Intuitively, one of the firms may occupy the exporting point to prevent the same location from being chosen by the rival. For simplicity, this study will only focus on symmetric solutions from now on. Combining the above two considerations yields the following proposition and corollaries.

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7 Note that the quantities in (4) and the prices in (5) depend on the distance between the market point and firms, and we thus need to rearrange the expression of distance after taking off the absolute signs. For example, $d(x, x_1)=t\cdot(x_1-x)$ when $x\in[0, x_1]$, while $d(x, x_1)=t\cdot(x-x_1)$ when $x\in[x_1, x_1+\frac{1}{2}]$.

Proposition 1. (1) *In a spatial circular market with a foreign market, there exists a critical point* F_{c_1} *such that when* $F \leq F_{c_1}$*, the only interior (separated) equilibrium locations are* $\left(x_1^* = \frac{2t + Ft + \sqrt{t^2(F+2)^2 - 32Ft(a - tK)}}{16t}, x_2^* = 1 - x_1^*\right)$*. In addition,* $\partial x_1^*/\partial F < 0$*,* $\partial x_2^*/\partial F > 0$*,* $\partial x_1^*/\partial K > 0$*,* $\partial x_1^*/\partial t > 0$*,* $\partial x_2^*/\partial t < 0$*, and* $\partial x_2^*/\partial K < 0$*.* (2) *An agglomerate location equilibrium exists when the foreign market is sufficiently large such that* $F \geq F_{c_2}$*, where* F_{c_2} *is a critical point,* $F_{c_1} \leq F_{c_2} \leq 1/2$*.*

The first part of Proposition 1 demonstrates that the relative size of the foreign market is crucial in determining the equilibrium locations. Intuitively, when F is small, firms will focus more on the domestic market, and thus the equilibrium locations are separated and are closer to the exporting point, compared with locating at the opposite ends of a diameter as in Pal (1998). When F is sufficiently large ($F > F_{c_1}$), the importance of the foreign market should be more heavily weighted by firms, inducing firms to locate at the exporting point, and so there is no pure strategy equilibrium with separated locations. It is not novel in trade theory that firms agglomerate if the foreign market is relatively large. However, Proposition 1 is a new result in the circular Cournot model with a foreign market.

The comparative statics shows that as F increases, more weights must be put on the foreign market, and so the equilibrium locations move closer to the exporting point. Additionally, when the transport rate increases, the weight of the foreign market is decreased, so the firms will locate further away from the exporting point. Finally, when the distance to the foreign market K increases, both firms have less incentives to locate near the exporting point.

The next corollary shows that, when $F > 0$, neither asymmetric interior locations ($x_1 \neq 1 - x_2$) nor asymmetric boundary locations, ($x_1 = 0$, $x_2 = 1/2$) or ($x_1 = 1/2$, $x_2 = 1$) are equilibria, which is consistent with most circular models.

Corollary 1. (1) *There does not exist any asymmetric interior location equilibrium.* (2) *Asymmetric boundary locations,* $(x_1, x_2) = (0, 1/2)$ *or* $(x_1, x_2) = (1/2, 1)$*, can never constitute an equilibrium when* $F > 0$*.*

The first part of Corollary 1 is intuitively clear, because we have symmetric setting of firms. For the second part, that both firms locate at the two ends of a diameter and one of them locates at the exporting point cannot be an equilibrium. Since there is a foreign market, the remotest location to the foreign market ($x_i = 1/2$) can never be the best response when its rival locates at the exporting point.

Corollary 2. *When* $F > 0$*, the interior (separated) locations are closer to the exporting point,* $x_1^* < 1/4$*,* $x_2^* > 3/4$*, compared with that without the foreign market. When*

F converges to zero, these equilibrium locations converge to a pattern which is consistent with the result of Pal (1998).

Since $x_1^*=1/4$, $x_2^*=3/4$ and $\partial x_1^*/\partial F<0$, when F converges to 0, we have $0<x_1^*<1/4$ when $F>0$. Notably, the interior location equilibrium never converges to $x_1^*=0$. In our model, when F converges to zero, the equilibrium locations are converge to $(x_1, x_2)=(1/4, 3/4)$, which is consistent with Pal (1998). However, we assume $F>0$ in this paper, thus the overall market is not uniform and $(x_1, x_2)=(0, 1/2)$ can never be an equilibrium. Only when $F=0$, as in Pal (1998), can (0, 1/2) constitute an equilibrium.

In contrast to the first part of Proposition 1 such that when the foreign market is small ($F<F_{c_1}$), these duopolistic firms will focus on the domestic market, and the only location equilibrium involves separated locations. The second part of Proposition 1 shows that if the foreign market is large ($F>F_{c_2}$), both firms agglomerate at the exporting point as a location equilibrium. Specifically, when $F\geq 1/2$, $(x_1=0, x_2=1)$ is always a location equilibrium. Propositions 1 yields the possibility that there does not exist pure strategy equilibrium when $F_{c_1}<F<F_{c_2}$. However, this range is very small by various numerical simulations. The above result indicates that an agglomeration equilibrium is very likely to exist. For example, when $a=tK+2t$, then $F_{c_1}=0.066$ and $F_{c_2}=0.086$. That is, if the size of the foreign market is more than 8.6 percent of the domestic market, then both firms agglomerate at the exporting point in equilibrium. It is worth noting that in a special case $K=0$, the exporting point can be seen as having a greater demand than all other points.[8] Then, this special case corresponds to an asymmetric circular model, and it is reasonable that firms move toward the exporting point in equilibrium.

IV. Social welfare for the domestic country

This section analyzes the socially optimal locations (x_1^o and x_2^o) for the domestic country. The total domestic consumer surplus is represented by

$$CS=\int_0^1 \frac{(a-p(x))\cdot(q_1(x)+q_2(x))}{2}\,dx, \tag{11}$$

where $(a-p(x))(q_1(x)+q_2(x))/2$ is the consumer surplus at location x. Calculations yield

8 We thank one of the referees for providing this point.

$$CS=\frac{1}{54b}(2t^2-6at+12a^2+6t^2(x_1-x_2)-18t^2x_1x_2+9t^2(x_1^2+x_2^2)$$
$$+12t^2x_1(x_2-x_1)+4t^2(x_1^3-x_2^3)). \quad (12)$$

The social welfare for the domestic country is defined as $W=CS+\Pi_1+\Pi_2$. Solving $\partial W/\partial x_1=0$ and $\partial W/\partial x_2=0$ simultaneously yields the social welfare maximization in the following proposition.[9]

Proposition 2. (1) *The socially optimal solution for the domestic country is* $\left(x_1^o=\frac{1}{56t}\left(7t+Ft+\sqrt{t^2(F+7)^2-112Ft(a-tK)}\right), x_2^o=1-x_1^o\right)$ *when* $F\leq F_{c_3}$, *and* $(x_1^o=0, x_2^o=1)$ *when* $F\geq F_{c_3}$, *where* F_{c_3} *is a critical point, with* $F_{c_3}>F_{c_2}$. (2) *The interior socially optimal solution locations of the two firms are closer to each other than those in the interior solution at equilibrium* ($x_1^o<x_1^*$) *when* $0<F<F_{c_1}$. (3) *The larger F is, the closer the interior socially optimal locations will be* $\left(\frac{\partial d(x_1^o, x_2^o)}{\partial F}<0\right)$.

Proposition 2 shows that the interior equilibrium locations are farther apart than the socially optimal locations for the domestic market. The socially optimal locations are suddenly flipped around $F=F_{c_3}$ and can be explained as follows. Since $(x_1^*=0, x_2^*=1)$ is a corner solution, it is socially more desirable than the interior solution (x_1^o, x_2^o) when F is large, and thus there is a discontinuous property around $F=F_{c_3}$. Matsumura and Shimizu (2005) show that in a linear market model, the duopolistic firms are more likely to either agglomerate or locate closer to each other than the socially optimal locations. In our circular market framework, the comparison between the equilibrium locations and the socially optimal locations depends on the size of the foreign market relative to that of the domestic market.

There are several cases depending on the value of F: when F converges to 0, both x_1^* and x_1^o converge to 1/4; when $0<F<F_{c_1}$, $x_1^o<x_1^*$ as shown in Proposition 2; when $F_{c_2}<F<F_{c_3}$, $x_1^o>x_1^*=0$; when $F\geq F_{c_3}$, $x_1^o=x_1^*=0$ as shown in Fig. 2. Intuitively, when F converges to 0, the model herein converges to a pure circular market as Pal (1998), and the equilibrium locations also converge to the social optimum where the total transportation costs are minimized.

Consider the case in which the foreign market is small ($0<F<F_{c_1}$). The foreign market has two opposite effects on x_1^* and x_1^o: whereas moving closer to the exporting point can reduce the costs of transportation to the foreign market (the cost-reducing effect), moving closer to each other also increases the competition in the domestic market (the competition effect). When the foreign market is small

9 There exists another solution in solving $\partial W/\partial x_1=0$ and $\partial W/\partial x_2=0$. However, it violates the second-order condition.

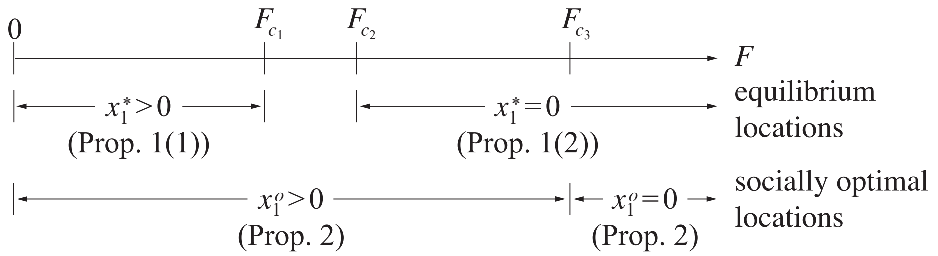

Fig. 2. The comparison between equilibrium locations and socially optimal locations

$(0<F<F_{c_1})$, firms tend to focus on the domestic market and the competition effect and so $x_1^o<x_1^*$. When $F_{c_2}<F<F_{c_3}$, firms are more likely to focus on the foreign market and the cost-reducing effect, and so $x_1^o>x_1^*=0$. When $F\geq F_{c_3}$, the foreign market is sufficiently large that the equilibrium locations and the socially optimal locations are both at the exporting point, and so $x_1^o=x_1^*=0$. In summary, the equilibrium locations may be either farther from or closer to each other than the socially optimal locations, and the equilibrium locations are socially desirable only when F converges to 0 or $F\geq F_{c_3}$. Compared with a linear market, as shown by Pal (1998), the consideration of saving transportation costs is dominated by the avoidance of competition in the circular market. Intuitively, the symmetric property in the circular market implies that every point has identical advantage in minimizing total transportation costs. In our framework, the exporting point minimizes the total transportation costs when F is large, due to the foreign market. Therefore, our framework indeed considers both the cost-reducing effect and the competition avoidance effect.

V. Mixed duopoly with a foreign market

Consider a scenario of mixed duopoly with one public firm and one private firm (denoted by firms 1 and 2, respectively). In the domestic market, suppose the public firm maximizes the domestic consumers' surplus and the total profits, while the private firm maximizes its own profit. Assume that both firms are profit maximizers in the foreign market.[10] Similarly, in the first stage both firms engage in location competition, and firms compete on quantity in the second stage.

For a domestic market point $x\in[0, 1]$, the profit functions are the same as equation (3). Following Matsushima and Matsumura (2003), the equilibrium quan-

10 If the public firm maximizes the joint profits in the foreign market, then it will set a monopoly price which may violate the anti-trust law. We rule out this case because it is rarely observed in practice.

tities at $x \in [0, 1]$ can be solved by $\partial W/\partial q_1 = 0$ and $\partial \pi_2/\partial q_2 = 0$, if $q_2(x) > 0$, and can be solved by $\partial W/\partial q_1 = 0$, if $q_2(x) = 0$:

$$q_1(x) = \frac{a - 2td(x, x_1) + td(x, x_2)}{b},$$
$$q_2(x) = \frac{t(d(x, x_1) - d(x, x_2))}{b}, \quad \text{if} \quad d(x, x_1) > d(x, x_2),$$

and

$$q_1(x) = \frac{a - td(x, x_1)}{b}, \quad q_2(x) = 0, \quad \text{if} \quad d(x, x_1) \leq d(x, x_2),$$

while the equilibrium price at x is $p(x) = td(x, x_1)$.[11] For the foreign market, firms' profits and equilibrium quantities are the same as (6) and (7). Similarly, the circular market can be divided into seven segments for further calculations: region I is $[0, x_1]$, region II is $[x_1, x_2 - \frac{1}{2}]$, region III is $[x_2 - \frac{1}{2}, \frac{x_1 + x_2}{2}]$, region IV is $[\frac{x_1 + x_2}{2}, x_1 + \frac{1}{2}]$, region V is $[x_1 + \frac{1}{2}, x_2]$, region VI is $[x_2, \frac{1 + x_1 + x_2}{2}]$, and region VII is $[\frac{1 + x_1 + x_2}{2}, 1]$ (see Fig. 3). Note that $q_2(x) = 0$ when $d(x, x_1) \leq d(x, x_2)$ (i.e. regions I, II, III, and VII). After some calculations, we can have the following proposition.

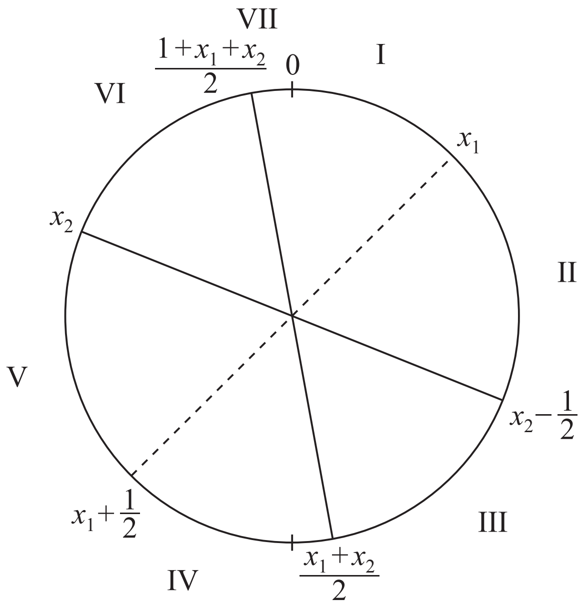

Fig. 3. The domestic market configuration under mixed duopoly

11 Since the public firm maximizes social welfare, firm 1 sets a price equal to the marginal cost $td(x, x_1)$. Therefore, firm 2 chooses zero output when his marginal cost $td(x, x_2)$ is higher than $td(x, x_1)$.

Proposition 3. *Consider a mixed duopoly case with one public firm and one private firm.* (1) *There exists a critical point* F_{c_4} *such that when* $F \le F_{c_4}$, *an equilibrium pair of locations is* $\left(x_1^* = \frac{1}{72t}\left(9t + 2Ft + \sqrt{t^2(2F+9)^2 - 288Ft(a-tK)}\right),\ x_2^* = \frac{1}{72t}\left(63t - 2Ft - \sqrt{t^2(2F+9)^2 - 288Ft(a-tK)}\right)\right)$. *Moreover,* $\partial x_1^*/\partial F < 0$, *and* $\partial x_2^*/\partial F > 0$. (2) *There exists a critical point* $F_{c_5} < 1/2$ *such that when* $F \ge F_{c_5}$, *a location pair* $(x_1 = 0, x_2 = 1)$ *constitutes an equilibrium.*

The above results show that Proposition 1 is robust when mixed duopoly is considered. Since (x_1^*, x_2^*) approach (1/4, 3/4) when F converges to 0 and $\partial x_1^*/\partial F < 0$, the interior solution satisfies $0 < x_1^* < 1/4$ and $3/4 < x_2^* < 1$, when $F > 0$. Compared with the case of Matsushima and Matsumura (2003), the interior solution will be closer to the exporting point when a foreign market is included.[12] The symmetric location equilibrium in the circular market with mixed duopoly is not rare. Matsushima and Matsumura (2003) obtained symmetric equilibrium locations in their Proposition 2 when there is only one private firm. Our result is consistent with the literature. In addition, when the relative size of the foreign market increases, the interior locations will move closer to the exporting point. Finally, the public firm and the private firm agglomerate at the exporting point when F is large. Specifically, $F > 1/2$ ensures the existence of an agglomeration equilibrium.

VI. Extension

This section provides three extensions, including two circular markets, export subsidies, and the competition between a domestic firm and a foreign firm.[13]

A. Location analysis with two circular markets

The models in Sections 2, 3, 4, and 5 treat the foreign market in a simplified manner, as a mere point. This setting is a reasonable approximation only when the distance between these two countries is relatively large. However, when these two countries are geographically close, the local transportation costs inside the foreign country should be measured precisely. Suppose the foreign country is also a circular city with one unit length, and there are F units of mass of consumers for any point, as shown in Fig. 4. In this subsection, firms can locate only in the domestic market.

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12 Matsushima and Matsumura (2003) consider a mixed oligopoly case with a public firm and many private firms, and show that the private firms will agglomerate at the point ($x = \frac{1}{2}$) opposite to the location ($x_1 = 0$) of the public firm.

13 The detailed derivations, formal propositions, and proofs are available from the authors upon request.

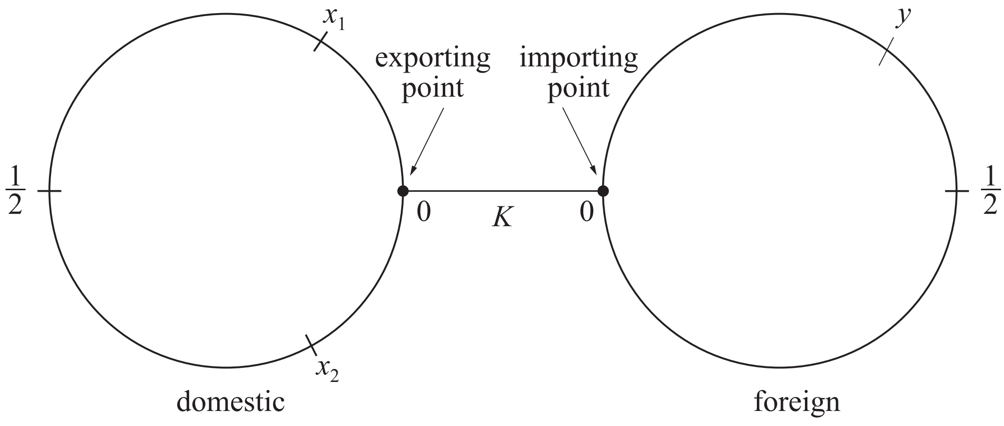

Fig. 4. The locational configuration under two circular markets

The domestic market is the same as the previous models, and so equations (1)-(5) are still valid. The local transportation costs in the foreign market should be counted by the distance between the importing point ($y=0$) and the local market $y\in[0, 1]$.[14] The solving process is similar to the models in Sections 2, 3, and 4. After some calculations, we have the following result, which is similar to Proposition 1. First, when the foreign country is also a circular market with one unit length, the interior equilibrium locations are $\left(x_1^*=\frac{2t+Ft+\sqrt{t^2(4+12F+F^2)-32Ft(a-tK)}}{16t},\ x_2^*=1-x_1^*\right)$, when F is small. Second, an agglomerate location equilibrium exists when F is large.

B. Export subsidies

In this subsection, an export policy on the previous framework with two circular markets is analyzed. Consider a per unit subsidy (s) to firms for their exports. Then, the profit functions are

$$\pi_1^f(y)=\left(a-\frac{b}{F}(q_1^f(y)+q_2^f(y))-t(x_1+K+\min\{y,\ 1-y\})+s\right)q_1^f, \tag{13}$$

$$\pi_2^f(y)=\left(a-\frac{b}{F}(q_1^f(y)+q_2^f(y))-t(1-x_2+K+\min\{y,\ 1-y\})+s\right)q_2^f. \tag{14}$$

Following similar calculations in the previous section, we have a similar result. With an export subsidy, the interior locations are closer to the exporting point, while the critical value for the existence of agglomeration location equilibrium is larger than the case without any export subsidy.

14 Note that in this case, the lower bound of a should be larger than $t(1+K)$ to ensure all markets are served by two firms.

This result shows that both firms will move closer to the exporting point when the subsidy is enacted. Intuitively, an export subsidy causes an effect similar to reducing the transport rate to the foreign market (or reducing the distance to the foreign market), so firms move closer to the exporting point to reflect the fact that more weights have been put on the foreign market. Our model contributes a spatial analysis to the non-spatial framework of duopolistic competition and export subsidies such as Brander and Spencer (1985) and others. Our results show that firms' locations would be changed with export subsidies.

C. The competition between a domestic firm and a foreign firm

In previous sections, our analysis is restricted to two domestic firms, without allowing firms to locate outside the domestic country. In this subsection we consider these two firms locating in different countries such that one domestic firm competes with one foreign firm. Without loss of generality, assume that firm 1 locates in country 1 (domestic country), and firm 2 locates in country 2 (foreign country) as shown in Fig. 5. The location of firm 1 is denoted by x_1, while the location of firm 2 is denoted by y_2, and $x_1 \in [0, 1/2]$, $y_2 \in [0, 1/2]$ without loss of generality. After comparing the interior and corner solutions, we have the following result. When firm 1 locates in the domestic market and firm 2 locates in the foreign market, (1) there exist no interior locations; (2) there are four possible corner solutions such that (2a) when a is large, a location pair $\{x_1^* = 0, y_2^* = 0\}$ is an equilibrium; (2b) when a is small, a location pair $\{x_1^* = 1/2, y_2^* = 1/2\}$ is an equilibrium; (2c) when a is moderate, a location pair $\{x_1^* = 0, y_2^* = 1/2\}$ constitutes an equilibrium when $F \geq 1$; while a location pair $\{x_1^* = 1/2, x_2^* = 0\}$ constitutes an equilibrium when $F \leq 1$.

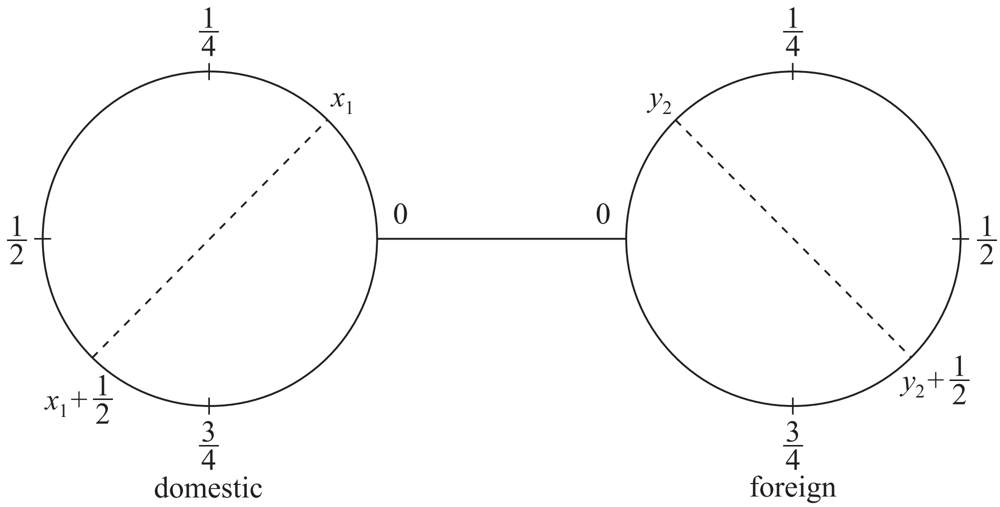

Fig. 5. The market configuration with one domestic firm and one foreign firm

It is intuitively clear that there is no incentive for these two firms to choose interior locations. For the corner solutions, it is straightforward that ($x_1=0$, $y_2=0$) is a location equilibrium when a is large, because both firms try to move closer to each other to take a larger market share. In contrast, when a is small, the location pair ($x_1=1/2$, $y_2=1/2$) is naturally an equilibrium, because they choose to locate at the weakest point of their rivals as per Pal (1998). When a is moderate, the locations of firms depends on the value of F. If F is large ($F\geq 1$), the domestic firm has an incentive to locate closer to the foreign market ($x_1=0$), and firm 2 will locate at $y_2=1/2$ to avoid competition. However, if F is small ($F<1$), then ($x_1=1/2$, $y_2=0$) is a location equilibrium.

Finally, our result can be extended to a more general case such that both firms can choose which country they wish to locate in. When F is sufficiently small, both firms will locate symmetrically at the domestic market with an equal distance to the exporting point. If F is moderate, both firms agglomerate at the exporting point, while if F is large but less than 1, firms will locate separately in different countries. This result also provides implications of foreign direct investments (FDI). Imagine that there are two domestic firms which have the options of FDI. Specifically, when F is small, both firms choose domestic interior locations; when F is intermediate, both firms agglomerate at the exporting point; when F is large, one firm chooses FDI, and the other firms locate at the exporting point; finally, when F is extremely large, both firms choose FDI.

VII. Conclusions

This study considers a framework with a location-then-quantity competition between duopoly firms in a circular market with an exporting point connected to a foreign market by a highway or a waterway. Our results show that there exists a separated solution of locations if the size of the foreign market is small. As the foreign market's size increases, the separated equilibrium locations move closer to the exporting point. If the size of the foreign market is large enough, then both firms agglomerate at the exporting point in equilibrium. The socially optimal separated locations are closer to each other than the separated equilibrium locations. Moreover, when the foreign market's size is relatively large, the equilibrium locations are socially desirable at the exporting point. We also show that the robustness of our findings can apply to the case of mixed duopoly.

An extensive framework with two circular markets and two domestic firms is also discussed. There exist interior locations when the size of the foreign market is small, while firms agglomerate at the exporting point when the foreign market is large. If one firm locates in the domestic market and the other firm locates in the foreign market, then there is no interior location equilibrium, and multiple corner solutions exist, depending on the reservation price and the relative size of the foreign market.

There are several policy implications for investment promotion from our study. First, to attract firms to cluster, in addition to expensive industrial parks arranged by governments, lowering export costs (erasing trade barriers) gives firms the same incentive to form industrial clusters. Secondly, enlarging the domestic market helps in keeping firms from migrating to foreign countries. Thirdly, export subsidies can also provide incentives for firms to move closer to the exporting point. In addition, either reducing trade barriers or providing export subsidies may attract foreign firms to engage in FDI in the domestic exporting area.

A natural extension of the present model would involve general multiple firms or multiple exporting points in a circular market. Another future line of research is to investigate a model of mixed oligopoly with multiple public and private firms. Intuitively, consider a simple extension on the case of three domestic firms. There may exist multiple location equilibria. When the size of the foreign market is small, one location equilibrium is constituted by separated locations with one firm being located at the exporting point. The other equilibrium could have two firms agglomerate at the exporting point, while the other firm is located separately. These two equilibria are similar to the results in Gupta et al. (2004). Additionally, when the size of the foreign market is large, there might exist an equilibrium where all firms agglomerate at the exporting point to benefit from saving transportation costs to the foreign market. The cases with more than three firms may be further explored.

Appendix

Proof of Proposition 1(1)

Simultaneously solving first-order conditions $\partial\Pi_1/\partial x_1=0$ and $\partial\Pi_2/\partial x_2=0$ yields the first solution

$$x_1^*=\frac{2t+Ft+\sqrt{t^2(F+2)^2-32Ft(a-tK)}}{16t},\quad x_2^*=1-x_1^*, \tag{A.1}$$

and the second solution

$$\hat{x}_1=\frac{2t+Ft-\sqrt{t^2(F+2)^2-32Ft(a-tK)}}{16t},\quad \hat{x}_2=1-\hat{x}_1. \tag{A.2}$$

However, the second solution violates the second-order condition, because

$$\left.\frac{\partial^2\Pi_1}{\partial x_1^2}\right|_{x_1=\hat{x}_1,\ x_2=\hat{x}_2}=\frac{2t\left(3Ft+\sqrt{t^2(F+2)^2-32F(a-tK)}\right)}{9b}>0. \tag{A.3}$$

For the first solution, the second-order condition requires

$$\left.\frac{\partial^2\Pi_1}{\partial x_1^2}\right|_{x_1=x_1^*,\ x_2=x_2^*}<0 \quad\text{and}\quad \left.\frac{\partial^2\Pi_2}{\partial x_2^2}\right|_{x_1=x_1^*,\ x_2=x_2^*}<0, \tag{A.4}$$

which yields

$$F<F_1=\frac{-(8a-t+8tK)+\sqrt{(8a-t+8tK)^2+8t^2}}{4t}. \tag{A.5}$$

Moreover, x_1^* is required to be a real number. Therefore, the critical values of F are

$$F_2=\frac{2\left(8a-t-8tK-4\sqrt{t^2K-ta+4a^2-8tKa+4t^2K^2}\right)}{t}, \tag{A.6}$$

$$F_3=\frac{2\left(8a-t-8tK+4\sqrt{t^2K-ta+4a^2-8tKa+4t^2K^2}\right)}{t}, \tag{A.7}$$

where $F_2<F_3$, such that either $F<F_2$ or $F>F_3$ is required. Differentiating x_1 with respect to F yields

$$\frac{dx_1^*}{dF}=\frac{\sqrt{t^2(F+2)^2-32Ft(a-tK)}+2t+Ft-16a+16tK}{16\sqrt{t(4t+4Ft+F^2t-32Fa+32FtK)}}. \tag{A.8}$$

Since the foreign market is not nil,

$$q_1^f+q_2^f=-\frac{\sqrt{t^2(F+2)^2-32Ft(a-tK)}+2t+Ft-16a+16tK}{24b}>0. \tag{A.9}$$

The numerator of (A.8) is identical to the numerator of (A.9) times a negative sign. Therefore, $dx_1^*/dF<0$. This means that as F increases, these equilibrium locations move closer to the exporting point if $F<F_2$ or $F>F_3$. However, $(q_1^f+q_2^f)|_{F=F_3}=-\frac{\sqrt{(tK-a)(4a-t-4tK)}}{3b}<0$. Therefore, the valid parameter region is $F<F_2$. Combining $F<F_1$ from the second-order condition and $F<F_2$ yields $F<\min\{F_1, F_2\}\equiv F_{c_1}$. Finally, $\partial x_1^*/\partial K=\frac{F\sqrt{t}}{\sqrt{(2t+F)^2-32Ft(a-tK)}}>0$. Similarly, $\partial x_1^*/\partial t>0$ is easily observed.

Proof of Proposition 1(2)

The necessary and sufficient conditions for the corner solution ($x_1=0$, $x_2=1$) are considered here. Whether any firm deviates from this corner solution must be checked. Suppose $x_2=1$, then the profit of firm 1 is

$$\Pi_1(x_1, x_2=1)=\frac{1}{108b}(12a^2(1+F)-6at(1+4F(K+2x_1)) +t^2(1+8(3-4x_1)x_1^2+12F(K+2x_1)^2)). \tag{A.10}$$

Clearly, $\Pi_1(x_1, x_2=1)$ is a third-order polynomial function in x, and the cubic term x_1^3 is negative. Therefore, we may find F_4 and F_5 such that $\pi_1(x_1, x_2=1)$ is a monotone decreasing function of x_1 when $F_4\leq F\leq F_5$, where

$$F_4=\frac{2a-t-2tK-2\sqrt{(a-tK)^2-t(a-tK)}}{2t}<\frac{1}{2},$$

$$F_5=\frac{2a-t-2tK+2\sqrt{(a-tK)^2-t(a-tK)}}{2t}>\frac{1}{2}.$$

It follows that ($x_1=0$, $x_2=1$) is an equilibrium if $F_4\leq F\leq F_5$.

When $F<F_4$ or $F>F_5$, $\Pi_1(x_1, x_2=1)$ should be a curve with one convex region and one concave region as shown in Fig. 6, where $x_1^{c_1}$ and $x_1^{c_2}$ are the solutions to $\partial\Pi_1(x_1, x_2=1)/\partial x_1=0$:

$$x_1^{c_1}=\frac{2Ft+t-\sqrt{t^2(1+2F)^2-8Ft(a-tK)}}{4t}, \tag{A.11}$$

$$x_1^{c_2}=\frac{2Ft+t+\sqrt{t^2(1+2F)^2-8Ft(a-tK)}}{4t}. \tag{A.12}$$

Differentiating $\Pi_1(x_1, x_2=1)$ with respect to x_1 yields the first-order derivative as

$$\frac{\partial\Pi_1(x_1, x_2=1)}{\partial x_1}=\frac{4}{9}\,\frac{t(-2tx_1^2+FtK-Fa+tx_1+2Fx_1)}{b},$$

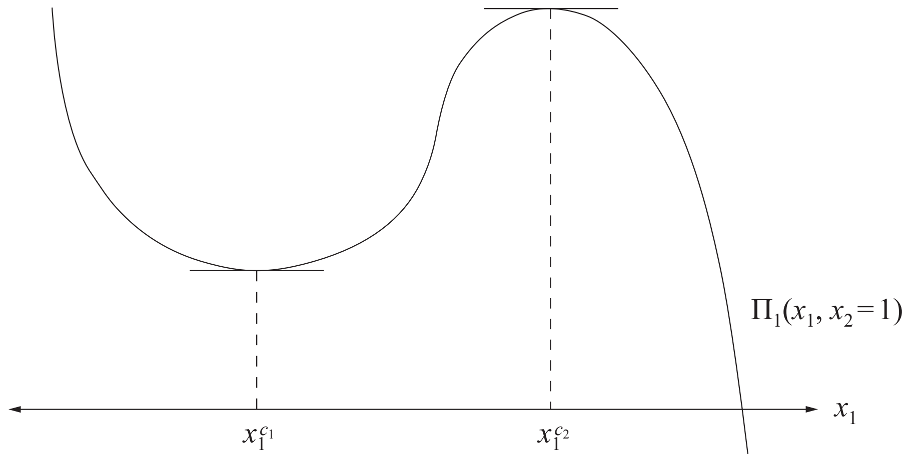

Fig. 6. Profit function $\Pi_1(x_1, x_2=1)$

and the second-order derivative as

$$\frac{\partial \Pi_1^2(x_1, x_2=1)}{\partial x_1^2}=\frac{4t(t-4tx_1+2Ft)}{9b}.$$

By assumption, $x_1 \in [0, 1/2]$. When $x_1=0$, $\frac{\partial \Pi_1(x_1=0, x_2=1)}{\partial x_1}=\frac{4t(FtK-Fa)}{9b}<0$. Similarly, when $x_1=1/2$, $\frac{\partial \Pi_1(x_1=1/2, x_2=1)}{\partial x_1}=\frac{4t(FtK-Fa+Ft)}{9b}<0$. Therefore, $x_1=0$ and $x_1=1/2$ are always in the segment of $\Pi_1(x_1, x_2=1)$ that has a negative slope. Since $\frac{\partial \Pi_1^2(x_1=0, x_2=1)}{\partial x_1^2}=\frac{4t(2Ft+t)}{9b}>0$, $x_1=0$ is located in the convex segment of $\Pi_1(x_1, x_2=1)$ and so x_1 is located to the left of $x_1^{c_1}$. The locations of $x_1=0$ and $x_1=1/2$ are described by two possible cases.

In Case 1, both $x_1=0$ and $x_1=1/2$ are located in $[0, x_1^{c_1}]$ as presented in Fig. 6. In Case 2, $x_1=0$ is located in $[0, x_1^{c_1}]$ and $x_1=1/2$ is located in $[x_1^{c_2}, 1]$. It is important to note that the relative locations of $(x_1^{c_1}, x_1^{c_2}, x_1=1/2)$ depend on F. In Case 1, $x_1=1/2<x_1^{c_1}$, while in Case 2, $x_1=1/2>x_1^{c_2}$. Since $\frac{\partial \Pi_1^2(x_1=1/2, x_2=1)}{\partial x_1^2}=\frac{4t(2Ft-t)}{9b}\gtrless 0$ if and only if $F\gtrless 1/2$, $x_1=1/2<x_1^{c_1}$, when $F\geq 1/2$, because $x_1=1/2$ is in the convex segment and $x_1=1/2>x_1^{c_2}$, when $F<1/2$.

When $F\geq 1/2$, $x_1=0$ and $x_1=1/2$ are both located left to $x_1^{c_1}$ (Case 1). Since $\Pi_1(x_1=0, x_2=1)>\Pi_1(x_1, x_2=1)$ for all $x_1\in[0, 1/2]$, $x_1=0$ is the best response to $x_2=1$. Similarly, $x_2=1$ can be shown to be the best response to $x_1=0$. Henceforth, $F\geq 1/2$ ensures that $(x_1=0, x_2=1)$ constitutes an equilibrium location.

Summarizing the above results shows that $(x_1=0, x_2=1)$ is an equilibrium when $F_4\leq F\leq F_5$ or $F\geq 1/2$. Therefore, $(x_1=0, x_2=1)$ is an equilibrium if $F\geq F_{c_2}\equiv F_4$, because $F_4 < 1/2$ and $F_5 > 1/2$. Moreover, $F_{c_2} > F_2$ by detailed calculations and so $F_{c_2} > F_{c_1}$. Moreover, $x_2 = 1$ and $x_1 \in [0, 1/2]$ will never be the solution. Given $x_2 = 1$, $\partial \Pi_1 / \partial x_1 = 0$ yields $\tilde{x}_1 = \frac{2Ft + t - \sqrt{(2F+1)^2 t^2 - 8Ft(a - tK)}}{4t}$ which violates the second-order condition, since

$$\left.\frac{\partial^2 \Pi_1}{\partial x_1^2}\right|_{x_1=\tilde{x},\, x_2=1} = \frac{4t\sqrt{t^2(2F+1)^2 - 8Ft(a-tK)}}{9b} > 0. \tag{A.13}$$

□

Proof of Corollary 1

(1) Since we do not employ a symmetric-location condition in solving the first-order conditions, $\partial \Pi_1 / \partial x_1 = 0$ and $\partial \Pi_2 / \partial x_2 = 0$, there does not exist any asymmetric interior solution of locations.

(2) In the case that $(x_1, x_2) = (1/2, 1)$, the first-order condition $\left.\frac{\partial \Pi_1}{\partial x_1}\right|_{x_1=\frac{1}{2},\, x_2=1} = \frac{-4Ft}{9b}(a - tk - t) < 0$ violates the boundary location solution required for firm 1. In the other case that $(x_1, x_2) = (0, 1/2)$, similarly, the first-order condition $\left.\frac{\partial \Pi_2}{\partial x_2}\right|_{x_1=0,\, x_2=\frac{1}{2}} = \frac{4Ft}{9b}(a - tk - t) > 0$, violates the boundary location solution required for firm 2.

□

Proof of Corollary 2

When F converges to zero, (x_1^*, x_2^*) will converge to (1/4, 3/4), which is equivalent to the result of Pal (1998). □

Proof of Proposition 2

Solving $\partial W / \partial x_1 = 0$ and $\partial W / \partial x_2 = 0$ yields

$$x_1^o = \frac{1}{56t}\left(7t + Ft + \sqrt{t^2(F+7)^2 - 112Ft(a - tK)}\right),$$
$$x_2^o = 1 - x_1^o. \tag{A.14}$$

The corner solutions to the social welfare maximization are obtained by using

$$\left.\frac{\partial W}{\partial x_1}\right|_{x_1=0,\, x_2=1} = -\frac{12Ft(a - tK)}{54b} < 0, \tag{A.15}$$

and

$$\left.\frac{\partial W}{\partial x_2}\right|_{x_1=0,\, x_2=1} = \frac{12Ft(a - tK)}{54b} > 0. \tag{A.16}$$

Therefore, $x_1=0$ and $x_2=1$ always constitute a corner solution. Comparing the social welfare between the corner solution and the interior solution yields

$$W(x_1=0, x_2=1)-W(x_1=x_1^o, x_2=1-x_1^o)>0 \quad \text{if} \quad F>F_{c_3}, \tag{A.17}$$

where

$$F_{c_3}=\frac{7}{3t}\left(32(a-tK)-3t-8\sqrt{(a-tK)(16(a-tK)-3t)}\right). \tag{A.18}$$

Therefore, the socially optimal location is the corner location only when $F>F_{c_3}$. By detailed calculations, we have $F_{c_3}>F_{c_2}$. Since $\frac{\partial x_1^o}{\partial F}<0$, the socially optimal (interior) location moves closer to the exporting point as F increases.

Comparing the socially optimal location and the interior equilibrium location x_1^o and x_1^* yields

$$\begin{aligned} x_1^o-x_1^*=\frac{-1}{112}\Big(&5Ft-2\sqrt{t^2(F+7)^2-112Ft(a-tK)} \\ &+7\sqrt{t^2(F+2)^2-32Ft(a-tK)}\Big)<0, \quad \forall \quad F>0, \end{aligned} \tag{A.19}$$

since $7\sqrt{t^2(F+2)^2-32Ft(a-tK)}>2\sqrt{t^2(F+7)^2-112Ft(a-tK)}$. □

Proof of Proposition 3

Similar to the proofs of Proposition 1, the objective function of firm 1 becomes

$$\begin{aligned} \Pi_1=&\int_0^{x_1}\frac{(a-t(x_1-x))^2}{2b}dx+\int_{x_1}^{x_2-\frac{1}{2}}\frac{(a-t(x-x_1))^2}{2b}dx+\int_{x_2-\frac{1}{2}}^{\frac{x_1+x_2}{2}}\frac{(a-t(x-x_1))^2}{2b}dx \\ &+\int_{\frac{x_1+x_2}{2}}^{x_1+\frac{1}{2}}\frac{3t^2(x-x_1)^2-4t^2(x-x_1)(x_2-x)+2t^2(x_2-x)^2+a^2-2at(x-x_1)}{2b}dx \\ &+\int_{x_1+\frac{1}{2}}^{x_2}\frac{3t^2(1-x+x_1)^2-4t^2(1-x+x_1)(x_2-x)+2t^2(x_2-x)^2+a^2-2at(1-x+x_1)}{2b}dx \\ &+\int_{x_2}^{\frac{1+x_1+x_2}{2}}\frac{3t^2(1-x+x_1)^2-4t^2(1-x+x_1)(x-x_2)+2t^2(x-x_2)^2+a^2-2at(1-x+x_1)}{2b}dx \\ &+\int_{\frac{1+x_1+x_2}{2}}^{1}\frac{(a-t(1-x+x_1))^2}{2b}dx+\pi_1^f. \end{aligned} \tag{A.20}$$

The profit of firm 2 is

$$\Pi_2=\int_{\frac{x_1+x_2}{2}}^{x_1+\frac{1}{2}}\frac{t^2(x_1+x_2-2x)^2}{b}dx+\int_{x_1+\frac{1}{2}}^{x_2}\frac{t^2(x_2-x_1-1)^2}{b}dx$$

$$+\int_{x_2}^{\frac{1+x_1+x_2}{2}} \frac{t^2(1-2x+x_1+x_2)^2}{b}dx+\pi_2^f. \tag{A.21}$$

It is noted that the profit of firm 2 ($\pi_2(x)$) is zero in regions I, II, III and VII, so the first-order conditions become

$$\frac{\partial \Pi_1}{\partial x_1}=\frac{9t^2(x_1-x_2+1)(2(x_1-x_2)+1)+4Ft(t(2x_2+x_1-2)+(a-tK))}{9b}, \tag{A.22}$$

$$\frac{\partial \Pi_2}{\partial x_2}=\frac{9t^2(2(x_1-x_2)^2+3(x_1-x_2)+1)-4Ft(t(2x_2+x_1-1)-(a-tK))}{9b}. \tag{A.23}$$

(1) The first-order conditions yield the symmetric interior solution in Proposition 3. In fact, substituting $x_2=1-x_1$ into the above two first-order conditions yields an identical equation

$$-\frac{2t(9tx_1-36tx_1^2+2FtK-2Fa+2Ftx_1)}{9b}=0.$$

This derivation confirms the symmetric property of the solution. Similarly, from the second-order conditions ($\partial^2\Pi_1/\partial x_1^2<0$, $\partial^2\Pi_2/\partial x_2^2<0$) and the requirement that x_1^* and x_2^* need to be positive real numbers, we can find the critical point:

$$\hat{F}_{c_4}=\min\left\{\frac{9}{16t}\left(t+7tK-8a+\sqrt{t^2(9+16K+64K^2)-16a(t+8tK-4a)}\right)\right.$$
$$\left.\frac{3}{8t}\left(8a-3t-8tK+4\sqrt{(a-tK)(4a-3t-4tK)}\right)\right\}.$$

Moreover, $\partial x_1^*/\partial F<0$ and $\partial x_2^*/\partial F>0$ are implied by $q_1^f>0$ and $q_2^f>0$, respectively.

(2) Given $x_2=1$, firm 1's profit function $\Pi_1(x_1, x_2=1)$ is also a third-polynomial function in x and the cubic term x_1^3 is negative. Therefore, we can find a critical point, $F_{c_5}=\frac{3}{8t}\left(8a-3t-8tK+4\sqrt{(a-tK)(4a-3t-4tK)}\right)>\frac{1}{2}$, such that $x_1=0$ is the maximizer of $\Pi_1(x_1, x_2=1)$ when $F\geq F_{c_5}$. Similarly, given $x_1=0$ for firm 2, $\Pi_2(x_2, x_1=0)$ has a maximum at $x_2=1$ when $F\geq F_{c_5}$.

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與外國市場相連的圓形市場之雙占區位選擇

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摘　要

本研究分析雙占廠商在一個具有出口點與外國市場相連的圓形市場進行數量競爭的問題。我們證明外國市場的相對大小對於廠商區位選擇具有決定性的影響。當外國市場很小時，存在一個分離的區位均衡。隨著外國市場的增大，分離的均衡區位會逐漸向出口點靠近。當外國市場很大時，兩廠商會聚集在出口點。我們的結果在混合雙占的情況下仍然適用。此外，均衡區位有可能比社會最適區位更靠近或更遠離。另外，我們也延伸探討了兩個圓形市場、出口補貼，及一家國內廠商與一家國外廠商的競爭等情境，最後我們提供了招商投資的相關經濟意涵。

關鍵字：區位、空間 Cournot 模型、圓形市場、出口補貼、外國市場

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